Auction 102 Assignment Phase Technical Guide

1 Introduction

This technical guide sets forth the details of the assignment phase, the second phase of the two-part clock auction adopted by the Commission for the 24 GHz Band. The assignment phase offers winners of generic spectrum blocks in the clock phase of Auction 102 the opportunity to bid for preferred specific frequency assignments. Using the procedures adopted in the *Auctions 101 and 102 Procedures Public Notice* and detailed below,[1] the bidding system will determine specific license assignments and any payment, above the final clock phase price, that a winning bidder will pay for the assignment.

The assignment phase is designed to promote two major goals. One of these is to make bidding relatively easy even though the underlying allocation problem is complex. The procedure promotes simplicity in several ways. First, to reduce the total number of bids that each bidder must make, it groups together non-top 40 PEAs within a region under certain conditions. Second, to simplify bidding strategy for bidders, it uses a *second-price* type of pricing rule that encourages bidders to bid according to their actual values for different blocks while ensuring that there is no bidder or group of bidders willing to pay more for an alternative feasible assignment. Third, a winner does not need to make any additional bids – or make any additional payments – in the assignment phase. This makes bidding easier not only in the assignment phase of the auction but in the clock phase as well, because bidders in the clock phase will know that they need not pay more for licenses than the prices bid in the clock phase.

A second, equally important goal is to promote efficient and intensive use of the spectrum. To achieve that, the assignment phase rules ensure that each bidder is assigned contiguous frequencies within each category in each PEA, even if the bidder does not participate in the assignment phase.

2 Assignment Rounds

The assignment phase consists of a series of *assignment rounds.* In each assignment round, licenses are assigned in up to six *assignment phase markets*, with each assignment phase market consisting of either a single PEA or a group of PEAs; see Section 2.1. Winning bidders from the clock phase that have a preference for specific license frequencies submit sealed bids for those licenses, separately for each category. Once an assignment round concludes, an optimization is solved for each category in each assignment phase market to assign specific frequency licenses to each winning bidder from the clock phase. Additional optimizations are solved to determine each bidder's assignment payment, which will be equal to or less than the bidder's bid value for the assignment.

The bidding system will determine whether to group PEAs into a single assignment phase market according to the rule detailed in Section 2.1 below.

[1] *See Auctions of Upper Microwave Flexible Use Licenses for Next-Generation Wireless Services*; *Notice and Filing Requirements, Minimum Opening Bids, Upfront Payments, and Other Procedures for Auctions 101 (28 GHz) and 102 (24 GHz); Bidding in Auction 101 Scheduled to Begin November 14, 2018*, Public Notice, 33 FCC Rcd 7575, 7644, para. 216 & n.406 (2018) (*Auctions 101 and 102 Procedures Public Notice*).

2.1 Grouping PEAs into a Single Assignment Phase Market

A set of PEAs P will be grouped into one *assignment phase market* if all of the following three conditions are met:

1. The PEAs in P are all in the same Regional Economic Area Grouping ("REAG")[2] and not in the top-40 PEAs;[3]
2. Either all PEAs in P are not subject to the small market bidding credit cap[4] or all PEAs in P are subject to the small market bidding credit cap; and
3. For each PEA in P, the supply of blocks is the same in each category[5] and the same bidders won the same number of blocks in each category.

Because of this grouping of PEAs, the number of assignment phase markets will be smaller than or equal to the number of PEAs.

<u>Example 1</u>: REAG Grouping

Suppose that PEA-060, PEA-069, and PEA-077 are all in REAG 1. These PEAs are not top-40 PEAs and do not have any incumbents in the 24 GHz band.

In each of these PEAs:

- Bidder #1 won one Category L block in the clock phase.
- Bidder #2 won one Category L block and two Category U blocks in the clock phase.
- Bidder #3 won three Category U blocks in the clock phase.

Then PEA-060, PEA-069, and PEA-077 will be grouped and treated as a single combined market for the assignment phase.

<u>Example 2</u>: Not Possible to Group

PEA-043 and PEA-045 are both in REAG 2 and are not in the top-40 PEAs. Suppose that neither of those PEAs has incumbents in the 24 GHz band. However, the winners in these two PEAs are as follows:

In PEA-043:

- Bidder #1 won one Category L block in the clock phase.
- Bidder #2 won one Category L block and two Category U blocks in the clock phase.
- Bidder #3 won three Category U blocks in the clock phase.

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[2] The six Regional Economic Area Groupings (REAG) are: Northeast, Southeast, Great Lakes, Mississippi Valley, Central, and West. Each of the remaining REAGs (*i.e.*, Alaska, Hawaii, Puerto Rico and US Virgin Islands, Guam and the Northern Mariana Islands, American Samoa, and the Gulf of Mexico) will be merged in one of the 6 main REAGs.

[3] The top-40 PEAs are PEAs 1 – 40.

[4] PEAs that are subject to the small market bidding credit cap are those PEAs with a population of 500,000 or less, which corresponds to PEAs 118–416, excluding PEA 412. *See Updating Part 1 Competitive Bidding Rules et al.*, Report and Order, Order on Reconsideration of the First Report and Order, Third Order on Reconsideration of the Second Report and Order, Third Report and Order, 30 FCC Rcd 7493, 7546-47, paras. 127-28 (2015).

[5] Because of this condition, an encumbered PEA cannot be grouped with a PEA that is not encumbered.

In PEA-045:

- Bidder #1 won one Category L block in the clock phase.
- Bidder #2 won one Category L block and two Category U blocks in the clock phase.
- Bidder #4 won three Category U blocks in the clock phase.

Then, PEA-043 and PEA-045 will not be grouped, because they do not have identical clock phase winners.

2.2 Sequencing of Assignment Rounds

The assignment phase begins with assignment rounds for top-40 PEAs (PEAs 1-40). The top-40 PEAs are ordered in descending order of pops, and bidding is conducted for a single assignment phase market per round, sequentially. Note that there is no grouping for the top-40 PEAs.

After bidding has been conducted for the top-40 PEAs, bidding is conducted simultaneously for the six REAGs, but in descending order of pops within each REAG.[6] That is, bidding may be conducted for up to six assignment phase markets at the same time, in order to speed up the assignment phase. The rounds continue until all assignment phase markets are assigned.

If an assignment phase market consists of multiple PEAs, its pops will be set to be equal to the sum of the pops of the PEAs that it comprises, for purposes of determining the sequencing.

Before bidding for the assignment phase starts, the bidding system will inform bidders about which PEAs have been grouped and the sequencing of assignment rounds.

The following tables show two examples of the sequencing of assignment phase markets. In the first example, there is no grouping, that is, each assignment phase market consists of a single PEA. In the second example, some assignment phase markets consist of multiple PEAs and, as a result, there are fewer assignment rounds.

Table 1: Sequencing of assignment phase markets with no grouping

Round	PEA					
1	001					
2	002					
…	…					
40	040					
	REAG 1	**REAG 2**	**REAG 3**	**REAG 4**	**REAG 5**	**REAG 6**
41	041	412	052	046	047	042
42	044	043	056	051	053	070
43	048	045	058	055	063	076
…	…	…	…	…	…	…

[6] With the exception of PEA 412, Puerto Rico, the PEA numbering is in descending order of pops.

Table 2: Sequencing of assignment phase markets with grouping

Round	PEA(s)					
1	001					
2	002					
…	…					
40	040					
	REAG 1	REAG 2	REAG 3	REAG 4	REAG 5	REAG 6
41	041; 044	412	052	046	047	042
42	048	043; 045	056	051; 055	053	070
43	060; 069; 077	050	058	059	063	076
…	…	…	…	…	…	…

As illustrated in the tables above, after bidding for the top-40 PEAs is finished, bidding for multiple assignment phase markets will be conducted in the same round.

3 Bidding

3.1 Bidding Options in PEAs Without Incumbents

For each assignment phase market and each category, the bidding system will determine all contiguous assignment options where the number of licenses is equal to the number of blocks that the winner has won in the clock phase per PEA in that category and assignment phase market.[7] This set is referred to as the *bidding options* of the bidder. Note that the bidding options of a bidder do not depend on the clock phase winnings of other bidders.[8]

Example 1: The bidder won one Category L block for each PEA of a given assignment phase market. Then, the bidder has two bidding options: A and B.

Example 2: The bidder won three Category U blocks for each PEA of a given assignment phase market. Then, the bidder has the following three bidding options: CDE, DEF, and EFG.

Example 3: The bidder won one Category L block and two Category U blocks for each PEA of a given assignment phase market. Then, the bidder will have a set of bidding options for each category.

For Category L, the two bidding options are: A and B.

For Category U, the four bidding options are: CD, DE, EF, and FG.

The bidder can bid on any of its bidding options. However, note that, in certain instances, some of the bidding options cannot be assigned. For instance, if another bidder won 3 blocks for Category U, the bidder of Example 3 above cannot be assigned DE *or* EF, because, if it were, it would not be possible to assign contiguous spectrum to the other bidder. The bid options of a bidder are not limited only to the options that can be won by the bidder, because limiting the bid options in that way may permit a bidder to infer the clock phase winnings of other bidders.

[7] In a PEA where there is a single block in Category UI, the bidding system will consider contiguity across Categories U and UI jointly in considering assignment options for a winner of the UI block and other U blocks.

[8] The three examples in this section assume unencumbered PEAs.

3.2 Automatic Assignments and Pre-Assignments

If a bidder has only one bidding option available in an assignment phase market for a category, the bidder will be automatically assigned the licenses in this option and will not be able to place bids for this assignment phase market. For example, a bidder that won two Category L blocks in a PEA will be automatically assigned AB.

If all winners in a PEA have only one bidding option available to each of them, then all licenses in that PEA are pre-assigned to winners and there will not be a round held for that PEA. Similarly, if a PEA has no winners from the clock phase (all blocks remained unsold), there will not be a round held for that PEA.

3.3 Bidding Rules

A bidder may specify a bid value for each bidding option that it is presented with for an assignment phase market. The bidder bids for a bidding option by specifying a non-negative whole dollar amount for that option.

A bidder that does not have clock phase winnings in a PEA will not have any bidding options in the corresponding assignment phase market and thus cannot submit bids for that market.

If an assignment phase market is a group of PEAs, then each bidder has the same clock phase winnings in each of those PEAs (because of the grouping rule described in Section 2.1). By specifying a bid value for a bidding option, the bidder indicates the maximum amount that it is willing to pay to be assigned that option in all those PEAs. A bidder will not be able to bid for different frequency assignments in the various PEAs within a group.

A winner in the clock phase is not required to bid in the assignment phase. In particular, such a bidder may not wish to bid if it is indifferent among all assignments that it may get. The bidding system will consider a bid value of zero for any set of blocks for which a bidder submits no bid.

3.4 Winning Assignments and Payments

After bidding in an assignment round concludes, the bids are processed to determine the winning assignments and the payments for that round. For each assignment phase market of the round, each bidder is then informed about its winning assignment and its assignment payment for each category in that assignment phase market. This information is disclosed to the bidder before the next assignment round starts.

4 Assignment Determination

For a given category in a given assignment phase market, an assignment is *feasible* if:

(1) Each bidder is assigned one of its bidding options; and

(2) Any unsold licenses (FCC-held) are contiguous.

The winning assignment for a given assignment phase market is determined separately for Category L and Category U by maximizing the sum of bid values across all bids for the category. Ties, if any, are broken by including pseudo-random numbers in an optimization.

Specifically, the assignment determination is done by solving two optimization problems for each category in each assignment phase market. The first optimization problem finds the maximum sum of bid values among all feasible assignments. The bidding system then solves another optimization problem using randomly generated numbers to break ties, if any. The solution to the latter optimization is selected as the final assignment.

To mathematically formulate the assignment determination, the following notation is used:

- N denotes the set of bidders in that assignment phase market and category, that is, the set of winners in this particular assignment phase market and this particular category from the clock phase.
- The FCC is referred to as bidder 0. $N \cup \{0\}$ is used to denote the set of bidders and the FCC.
- K denotes the set of blocks for the category. K consists of Blocks A and B when the category is L. K consists of Blocks C, D, E, F, and G when the category is U.
- S denotes a set of blocks. For each block k, S_k denotes the indicator variable of whether block k is in set S. That is, $S_k = 1$ if $k \in S$, and $S_k = 0$ if $k \notin S$.
- m_j is the number of blocks per PEA won by bidder j in the assignment phase market and category of interest.
- m_0 is the number of unsold blocks per PEA in the assignment phase market and category of interest.
- F_j denotes the set of bidding options for bidder j. That is, F_j will consist of all sets $S \subseteq K$ with m_j contiguous licenses (see Section 3 for examples).
- F_0 consists of all sets $S \subseteq K$ with m_0 contiguous licenses. This set gives the possible assignments for any unsold licenses.
- $b_j(S)$ denotes the bid value of bidder j for set $S \in F_j$.
- b denotes the set of bid values.

Variable Definition:

$X_j(S)$ is a binary decision variable which has a value of 1 if exactly the blocks of set S are assigned to bidder j and 0 otherwise. This variable is defined for all $j \in N \cup \{0\}$. Thus, $X_0(S) = 1$ if the set of licenses assigned to the FCC is S.

4.1 Maximum Sum of Bid Values

$$r(b) = max \sum_{j \in N} \sum_{S \in F_j} b_j(S) \cdot X_j(S)$$

Subject to:

$$\sum_{j \in N \cup \{0\}} \sum_{S \in F_j} S_k \cdot X_j(S) = 1 \qquad \forall k \in K \qquad (1)$$

$$\sum_{S \in F_j} X_j(S) = 1 \qquad \forall j \in N \cup \{0\} \qquad (2)$$

$$X_j(S) \in \{0,1\} \qquad \forall j \in N \cup \{0\}, \forall S \in F_j \qquad (3)$$

The objective function is equal to the sum of bid values of an assignment, across all bidders.

Explanation of Constraints:

- Constraint (1) ensures that each block is assigned exactly once, either to one of the bidders or to the FCC.
- Constraint (2) ensures that each bidder is assigned exactly one of its bidding options and that the set of licenses assigned to the FCC is contiguous.
- Constraint (3) states that each decision variable $X_j(S)$ can be either equal to 0 or 1.

4.2 Tie-breaking

For every set S and every bidder $j \in N$, the bidding system generates a pseudo-random number $\xi_j(S)$ drawn uniformly at random from the set $\{1,2,\ldots,10^8\}$. The bidding system then solves an optimization problem to find the assignment that maximizes the sum of pseudorandom numbers among all assignments that satisfy constraints (1) through (3) of Section 4.1 such that the sum of bid values is equal to $r(b)$. In particular, the optimization problem is formulated as follows:

$$max \sum_{j \in N} \sum_{S \in F_j} \xi_j(S) \cdot X_j(S)$$

Subject to:

$\sum_{j \in N \cup \{0\}} \sum_{S \in F_j} S_k \cdot X_j(S) = 1$	$\forall k \in K$	(1)
$\sum_{S \in F_j} X_j(S) = 1$	$\forall j \in N \cup \{0\}$	(2)
$X_j(S) \in \{0,1\}$	$\forall j \in N \cup \{0\}, \forall S \in F_j$	(3)
$\sum_{j \in N} \sum_{S \in F_j} b_j(S) \cdot X_j(S) \geq r(b)$		(4)

Constraints (1) through (3) are the same as in the optimization of Section 4.1.

Explanation of New Constraint:

Constraint (4) states that the sum of bid values must be greater than or equal to the result of the optimization of Section 4.1.

5 Assignment Payment Determination

The assignment payment that the bidder will pay for the set of licenses it is assigned for a category in an assignment phase market is an additional payment amount above the final clock phase prices before bidding credits are applied. If a bidder did not bid (or placed a bid of zero) for the set of licenses that it is assigned, then no additional calculation is necessary, and the bidder will not have any additional assignment payment for that category in that assignment phase market. If, on the other hand, the bidder placed a positive bid for the winning assignment, then the bidding system will calculate a type of 'second-price' assignment payment.[9]

The bidding system will apply bidder-optimal core prices and will use the "nearest Vickrey" approach to determine the assignment payments. In some cases, for Category U, the second price (Vickrey price) may not be high enough to ensure that no bidder or group of bidders is willing to pay more for an alternative feasible assignment, and so an additional payment above Vickrey prices may be required. In the event that such a payment is required, the calculation of the additional payment to be paid by each bidder will be weighted based on the number of blocks won by the bidder in the clock phase for Category U in that assignment phase market.

Such an additional payment will not be required for Category L. This is a consequence of the fact that Category L consists only of two blocks. That is, for Category L, each bidder will always pay its Vickrey price.

The assignment payments will satisfy the following conditions:

First condition: Each assignment payment must be positive or zero and not more than the dollar amount of the winning assignment phase bid.

[9] In some cases, this may also be zero.

Second condition: The set of assignment payments must be sufficiently high that there is no bidder or group of bidders willing to pay more for an alternative feasible assignment. If there is only one set of assignment payments that satisfies the first two conditions, this determines the assignment payments.

Third condition: If there are many sets of assignment payments that fulfil the first and second conditions, the set(s) of assignment payments minimizing the sum of assignment payments across all bidders is (are) selected. If there is only one set of assignment payments that satisfies these three conditions, this determines the assignment payments.

Fourth condition: If there are many sets of assignment payments that satisfy the first three conditions, the set of assignment payments that minimizes the weighted sum of squares of differences between the assignment payments and the Vickrey prices will be selected. The weighting is relative to the number of Category U blocks won by the bidder in the assignment phase market. This approach for selecting among sets of assignment payments that minimize the sum of assignment payments across bidders is referred to as the "nearest Vickrey" approach.

Section 5.1 describes how the Vickrey prices are calculated. Section 5.2 describes how payments for Category U are adjusted (if needed) to ensure that there is no bidder or group of bidders willing to pay more for an alternative feasible assignment. Section 5.3 provides an example.

5.1 Vickrey Price Calculation

For each bidder, the bidding system will determine the Vickrey price by re-solving the optimization problem of Section 4.1, but setting all bids of the bidder to zero while keeping the bids of every other bidder unchanged from the prior optimization, and calculate a hypothetical maximum sum of bid values from that optimization. The difference between the maximum sum of bid values associated with the actual optimization and the hypothetical maximum sum of bid values that would occur had that bidder provided all bids of zero will indicate the amount by which the bidder's winning bid amount exceeded the minimum amount it would have needed to bid to ensure the same winning assignment set. The Vickrey price is calculated by subtracting that amount from the bidder's actual bid amount.

Specifically: Let $r(b)$ denote the maximum value attained by solving the optimization problem of Section 4.1 when the set of bid values is b. Let b_j^* be the bid amount of bidder j for the bidding option that it is assigned.

The Vickrey price of bidder j for a given category in a given assignment phase market is:

$$p_j^{Vickrey} = b_j^* - \left(r(b) - r\left(b_{j\to 0}\right) \right)$$

where $b_{j\to 0}$ represents the set of bid values where the bid values of all bids of bidder j are set to zero (and the bid values of every other bidder are not changed).

For Category L, a bidder's assignment payment is equal to its Vickrey price.

5.2 Core Adjustments (Applicable only to Category U)

An extra payment beyond the Vickrey prices is sometimes required for Category U in order to satisfy the second condition, which requires that the set of assignment payments is sufficiently high that there is no bidder or group of bidders prepared to pay more for an alternative feasible assignment. A bidder or group of bidders willing to pay more for an alternative feasible assignment is referred to as a blocking coalition of bidders. The group which is willing to pay the most forms the first blocking coalition. A blocking coalition is unblocked by increasing the assignment payments while ensuring that each bidder's assignment payment is less than or equal to the corresponding bid amount. After adjusting the assignment payments to unblock the first blocking coalition, additional blocking coalitions may exist which are each unblocked by again increasing the assignment payments until there is no bidder or group of bidders willing to pay more for an alternative feasible assignment. Each bidder's assignment payment is guaranteed to be at least its Vickrey price and no more than its bid amount for its assignment.

Assignment payments can be calculated iteratively via a core adjustment process. This process operates by starting with Vickrey prices and then by iteratively adjusting assignment payments until there is no bidder or group of bidders willing to pay more for an alternative feasible assignment. It does so by gathering pricing constraints from each blocking coalition and then satisfying the pricing constraints by selecting assignment payments which minimize the distance, weighted by the number of Category U blocks won by each bidder in the given assignment phase market, from Vickrey prices.

To mathematically formulate the core adjustment calculations, the following notation is used in addition to the notation of Section 4:

- b_j^* is the bid amount of bidder j for the bidding option that it is assigned (in the original assignment determination problem).
- p_j^n is the payment of bidder j for core adjustment in iteration n.
- C^n denotes the blocking coalition for iteration n.

In the first iteration, p_j^1 is set to be equal to the Vickrey price of bidder j, that is, $p_j^1 = p_j^{Vickrey}$.

Given a set of assignment payments for iteration n, calculate a reduced bid for each bidder and each of its bidding options by deducting the current surplus of the bidder ($b_j^* - p_j^n$) from the corresponding bid amount. Specifically, in iteration n, the reduced bid of bidder j for a bidding option $S \in F_j$ is:

$$b_j^n(S) = \max\left\{b_j(S) - \left(b_j^* - p_j^n\right), 0\right\}$$

Let C^n be the set of bidders with a strictly positive winning bid in the solution of the optimization problem of Section 4.1 when solving the assignment determination problem for the set of reduced bids b^n. These bidders form the potential blocking coalition for iteration n. Among all potential blocking coalitions for iteration n, C^n is the one with the highest value, that is, $r(b^n)$.

If the value of the potential blocking coalition for iteration n does not exceed the sum of assignment payments for iteration n (that is, if $r(b^n) \leq \sum_{j \in N} p_j^n$), then there is no blocking coalition for iteration n and p_j^n represents the assignment payment for bidder j. In this case, the assignment payments have been determined and no further calculations are required.

Otherwise, bidders in C^n do form a blocking coalition for iteration n, and the bidding system will calculate the updated set of assignment payments p_j^{n+1} as described below.

The bidding system will first calculate the minimum sum of assignment payments required to unblock all coalitions as of iteration n. This is done by solving the following optimization problem:

Minimize Sum of Assignment Payments (for third condition)

$$\mu^n = \min \sum_{j \in N} p_j$$

Subject to:

$\sum_{j \in N \setminus C^i} p_j \geq r\left(b^i\right) - \sum_{j \in C^i} p_j^i$	$\forall i \in \{1, \dots, n\}$	(1)
$p_j \geq p_j^{Vickrey}$	$\forall j \in N$	(2)
$p_j \leq b_j^*$	$\forall j \in N$	(3)

Explanation of Constraints:

- Constraint (1) ensures that all coalitions are unblocked, that is, for the blocking coalition of each iteration i, the sum of assignment payments by bidders not in the coalition must be greater than or equal to the value of the coalition under the set of reduced bids for iteration i less the total iteration i payments of bidders in that coalition.
- Constraint (2) requires that the price paid by each bidder be greater than or equal to the bidder's Vickrey price.
- Constraint (3) requires that the price paid by each bidder be less than or equal to the bidder's bid amount for its winning assignment.

The bidding system will then update the assignment payments of bidders such that they sum up to μ^n (the third condition).

If there is more than one set of assignment payments that sum up to μ^n, the set of assignment payments that minimizes the weighted sum of squares of differences between the assignment payments and the Vickrey prices will be selected. The weighting is relative to the number of Category U blocks that the bidder won in that assignment phase market (the fourth condition).

The updated assignment payments p_j^{n+1} are calculated as the optimal solution to:

Minimize Distance Between Assignment Payments and Vickrey Prices (for fourth condition)

$$\min \sum_{j \in N} \frac{(p_j^{n+1} - p_j^{Vickrey})^2}{m_j}$$

Subject to:

$\sum_{j \in N \setminus C^i} p_j^{n+1} \geq r(b^i) - \sum_{j \in C^i} p_j^i$	$\forall i \in \{1, \dots, n\}$	(1)
$p_j^{n+1} \geq p_j^{Vickrey}$	$\forall j \in N$	(2)
$p_j^{n+1} \leq b_j^*$	$\forall j \in N$	(3)
$\sum_{j \in N} p_j^{n+1} = \mu^n$		(4)

This quadratic problem minimizes the weighted sum of squares of differences between the updated assignment payments p_j^{n+1} and the Vickrey prices $p_j^{Vickrey}$, weighted by the number of Category U blocks (m_j) won by each bidder.

Constraints (1) through (3) are the same as in the previous optimization.

Explanation of New Constraint:

Constraint (4) ensures that the sum of the updated payments is equal to the minimum amount required to unblock all of the coalitions up to iteration n.

5.3 Example

This section provides an example to illustrate how Vickrey prices and core adjustments are calculated in order to determine the assignment payments. In this example, Bidder 1 won one Category U block, and Bidders 2 and 3 won two Category U blocks each. The bids of each bidder are shown in Table 3.

Table 3: Assignment Payment Calculation Example

Bidders	*Bidder 1 (1 block)*	*Bidder 2 (2 blocks)*	*Bidder 3 (2 blocks)*
Bids	**C: $0**	CD: $0	CD: $0
	D: $0	**DE: $2,000**	DE: $0
	E: $0	EF: $0	EF: $0
	F: $0	FG: $0	**FG: $3,000**
	G: $1,000		
Vickrey prices (p^1)	$0	$0	$0
Bidder Surplus at p^1	$0	$2,000	$3,000
Reduced Bids Iteration 1 (b^1)	C: $0	CD: $0	**CD: $0**
	D: $0	DE: $0	DE: $0
	E: $0	**EF: $0**	EF: $0
Coalition	F: $0	FG: $0	FG: $0
$C^1 = \{1\}$	**G: $1,000**		
Adjusted payments p^2 (Assignment payments)	$0	$500	$500
Bidder Surplus at p^2	$0	$1,500	$2,500
Reduced Bids Iteration 2 (b^2)	**C: $0**	CD: $0	CD: $0
	D: $0	**DE: $500**	DE: $0
	E: $0	EF: $0	EF: $0
No blocking	F: $0	FG: $0	**FG: $500**
coalition	G: $1,000		

Assignment Determination. The sum of bid values is maximized when Bidder 1 is assigned C, Bidder 2 is assigned DE, and Bidder 3 is assigned FG. The value of this assignment is equal to $5,000.

Vickrey Prices. In this example, all Vickrey prices are $0 even though Bidder 3 is assigned a block for which Bidder 1 bid $1,000.

- The Vickrey price of Bidder 1 is equal to $0, because its bid value for its assignment is $0.
- To calculate the Vickrey price of Bidder 2, the bidding system would solve the assignment determination optimization problem with all of the bids of Bidder 2 set to $0. The optimal value of this optimization problem is $3,000, because Bidder 3 would be assigned FG (with a value of $3,000) and the remaining blocks would be assigned to Bidders 1 and 2 (with a value of $0). Thus, the Vickrey discount for Bidder 2 is equal to $5,000-$3,000 = $2,000. The Vickrey price of Bidder 2 is then calculated as its bid amount for its assignment ($2,000) less its Vickrey discount ($2,000) and is therefore equal to $0.
- To calculate the Vickrey price of Bidder 3, the bidding system would solve the assignment determination optimization problem with all of the bids of Bidder 3 set to $0. The optimal value of this optimization problem is $2,000, because Bidder 1 would be assigned C (with a value of $0), Bidder 2 would be assigned DE (with a value of $2,000) and Bidder 3 would be assigned blocks FG (with a value of $0). Note that it is not feasible to assign G to Bidder 1 and DE to Bidder 2, because then it would not be possible to assign contiguous spectrum to Bidder 3. Thus, the Vickrey discount for Bidder 3 is equal to $5,000-$2,000 = $3,000. The Vickrey price of

Bidder 3 is then calculated as its bid amount for its assignment ($3,000) less its Vickrey discount ($3,000) and is therefore equal to $0.

Iteration 1. The next step is to determine whether there is a blocking coalition for iteration 1. To do this, the bidding system would first calculate the surplus of each bidder for the first iteration as the bidder's bid amount for its assignment less its Vickrey price. The surplus is $0 for Bidder 1, $2,000 for Bidder 2, and $3,000 for Bidder 3. The bidding system would then calculate a set of reduced bids for each bidder by subtracting the bidder's surplus from its actual bid amount for each bidding option. If the result for a bidding option is negative, the reduced bid amount is set to be equal to $0. The reduced bid amounts for the first iteration, b^1,are shown in Table 3. The bids of Bidder 1 are not reduced because Bidder 1 derives $0 surplus. The bids of Bidder 2 are reduced by up to $2,000 while ensuring that all bid values are non-negative. Similarly, the bids of Bidder 3 are reduced by up to $3,000 while ensuring that all bid values are non-negative.

The assignment determination problem is solved with the set of reduced bids for iteration 1. There is a blocking coalition (consisting of Bidder 1) with value $1,000. The bidding system then calculates that the minimum sum of assignment payments to unblock this coalition is $\mu^1 = 1{,}000$. Because in this example Bidders 2 and 3 won the same number of blocks, the assignment payment of each of those bidders will be incremented by the same amount, namely, by $500. Thus, the assignment payments for iteration 2 are: $p_1^2 = 0$ and $p_2^2 = p_3^2 = 500$.

Iteration 2. The bidding system checks whether there exists another blocking coalition by solving the assignment determination problem for the reduced set of bids b^2 shown in Table 3. The maximum sum of bids is equal to $1,000 which does not exceed the sum of assignment payments for this iteration. Thus, there does not exist a blocking coalition in iteration 2. This implies that the assignment payments are equal to p^2, that is, $0 for Bidder 1, $500 for Bidder 2 and $500 for Bidder 3.

6 Final Auction Payment

When all assignment rounds have completed, a bidder's final payment is determined by summing the final clock phase prices across all licenses that it won and its assignment payments across all assignment phase markets, and then applying any applicable bidding credit discounts to the sum.

This section uses the following notation:

- M denotes the set of assignment phase markets
- C denotes the set of license categories
- SM denotes the set of assignment phase markets that consist solely of PEAs subject to the small market bidding cap.[10]
- BC denotes the bidder's bidding credit percentage (*e.g.*, 25 percent)
- $GP_{k,c}$ denotes the bidder's gross payment for category c in assignment phase market k. This is equal to the sum of the final clock phase prices for all licenses in the bidder's assignment *and* the bidder's assignment payment, for category c in assignment phase market k.
- FGP denotes the bidder's final gross payment

[10] Note that according to the rules for grouping PEAs into a single assignment phase market described in Section 2.1, *either* all PEAs in an assignment phase market are subject to the small market bidding credit cap *or* none of the PEAs in that assignment phase market are subject to the small markets bidding credit cap.

A bidder's final gross payment is calculated as the sum of its gross payments across all assignment phase markets and categories:

$$FGP = \sum_{k \in M} \sum_{c \in C} GP_{k,c}$$

For a bidder that does not qualify for any bidding credits, the final auction payment is equal to its final gross payment.

For a bidder that qualifies for the rural service provider bidding credit, the final (net) auction payment is:

$$FGP - \min\{\$10 \text{ million}, BC \cdot \text{FGP}\}$$

That is, to compute the applicable discount, the bidder's final gross payment is multiplied by the bidder's bidding credit and capped at $10 million.

For a bidder that qualifies for the small business bidding credit, the final (net) auction payment is:

$$FGP - \min\left\{\$25 \text{ million}, BC \cdot \sum_{k \in M \setminus SM} \sum_{c \in C} GP_{k,c} + \min\left\{\$10 \text{ million}, BC \cdot \sum_{k \in SM} \sum_{c \in C} GP_{k,c}\right\}\right\}$$

The calculation first caps the discount from small markets at $10 million, then adds the discount from all other markets and caps the total at $25 million.

7 Calculations for Payment Information During Assignment Phase

While winning bidders will be expected to pay the final auction payment set forth immediately above, during the assignment phase, the bidding system will show each bidder information intended to give the bidder a running estimate of its auction payment obligations. After each assignment round, each bidder will be shown its current gross total payment. A bidder that qualifies for a bidding credit will also be shown the net total payment as well as the corresponding capped and uncapped discounts. This way a bidder will know if it has reached any applicable bidding credit caps and the amount by which it is under or over. Moreover, this information will provide the bidder a running estimate during the assignment rounds of the dollar amount it will owe at the end of the auction.

In addition to the notation of Section 6, the following notation is used to give formulas for how the current total payments and the corresponding capped and uncapped bidding credit discounts are calculated for a given bidder:

- $FCP_{j,c}$ denotes the final clock phase price for Category c in PEA j
- $Q_{j,c}$ denotes the number of blocks won by the bidder for Category c in PEA j in the clock phase
- $AP_{k,c}$ denotes the bidder's assignment payment for category c in assignment phase market k
- A denotes the set of assignment phase markets for which an assignment has been processed. Note that A increases after every round.
- $GTP(S)$ denotes the bidder's gross total payment when S is the set of assignment phase markets for which an assignment has been processed.

All the discount calculations described in the following sections will be rounded to the nearest dollar.

7.1 Gross Total Payment

A bidder's *gross total payment* $GTP(S)$ is calculated as:

$$GTP(S) = \sum_{j=1}^{416} \sum_{c \in C} FCP_{j,c} \cdot Q_{j,c} + \sum_{k \in A} AP_k$$

The first term, which represents the clock phase payment, is the product of the final clock phase price and the number of blocks won by the bidder, summed over all 416 PEAs and all categories. The second term is the sum of the bidder's assignment payments across all assignment phase markets that have been assigned so far.

7.2 Rural Service Provider Bidding Credit

In addition to its gross total payment, a bidder that qualifies for the rural service provider bidding credit is shown the corresponding uncapped discount, capped discount, and net total payment, after each assignment round.

The *uncapped total payment discount* is:

$$BC \cdot GTP(S)$$

The *capped total payment discount* is:

$$min\{\$10 \text{ million}, BC \cdot GTP(S)\}$$

That is, the bidder's gross total payment is multiplied by its bidding credit percentage and capped at $10 million.

The *net total payment* is equal to the bidder's gross total payment minus its capped total payment discount. Once all assignment rounds have been processed, a bidder's net total payment is equal to its final (net) auction payment.

7.3 Small Business Bidding Credit

In addition to its gross total payment, a bidder that qualifies for the small business bidding credit is shown the corresponding uncapped discount in small markets, uncapped discount, capped discount, and net total payment.

In this section, $GTP_{SM}(S)$ is used to denote the part of the gross total payment that relates to PEAs subject to the small market bidding credit cap and $GTP_{NSM}(S)$ is used to denote the part of the payment that relates to PEAs not subject to the small market bidding credit cap.

The *uncapped total payment discount for small markets* only is:

$$BC \cdot GTP_{SM}(\mathrm{S})$$

The *uncapped total payment discount* is:

$$BC \cdot GTP(S)$$

The *capped total payment discount* is:

$$min\{\$25 \text{ million}, BC \cdot GTP_{NSM}(S) + min\{\$10 \text{ million}, BC \cdot GTP_{SM}(S)\}\}$$

This calculation first caps the discount from small markets at $10 million, then adds the discount from all other markets and caps the total at $25 million.

The *net total payment* is equal to the bidder's gross total payment minus its capped total payment discount. Once all assignment rounds have been processed, a bidder's net total payment is equal to its final (net) auction payment.

8 Post-Auction per-License Price Calculations

While final auction payments for winning bidders will be calculated as in Section 6 above, with bidding credit caps and assignment payments applying on an aggregate basis, rather than for individual licenses, the bidding system will also calculate a per-license price for each license. Such individual prices may be needed in the event that a licensee subsequently incurs license-specific obligations, such as unjust enrichment payments.

After the assignment phase, the bidding system will determine a net and gross post-auction price for each license that was won by a bidder by apportioning assignment payments and bidding credit discounts (only applicable for the net price) across all the licenses that the bidder won. To calculate the gross per-license price, the bidding system will apportion the assignment payment to licenses in proportion to the final clock phase price of the licenses that the bidder is assigned in that category and market. To calculate the net price, the bidding system will first apportion any applicable bidding credit discounts to each category and assignment phase market in proportion to the gross payment for that category and that market. Then, for each assignment phase market, the bidding system will apportion the assignment payment and the discount to licenses in proportion to the final clock phase prices of the licenses that the bidder is assigned in that category for that market.

In addition to the notation of Sections 6 and 7, the following notation will be used in this section:

- L_k denotes the set of licenses that the bidder was assigned in assignment phase market k
- TD denotes the total discount of the bidder
  - If the bidder qualifies for the rural service provider bidding credit:

$$TD = min\{\$10 \text{ million}, BC * FGP\}$$

  - If the bidder qualifies for the small business bidding credit:

$$TD = min\left\{\$25 \text{ million}, BC \cdot \sum_{k\in M\setminus SM}\sum_{c\in C} GP_{k,c} + min\left\{\$10 \text{ million}, BC \cdot \sum_{k\in SM}\sum_{c\in C} GP_{k,c}\right\}\right\}$$

8.1 Apportioning Discounts to Each Category in Each Assignment Phase Market

This section describes how to apportion the bidder's total bidding credit discount across assignment phase markets. Let $D_{k,c}$ denote the discount that is apportioned to assignment phase market k and category c.

If the bidder does not qualify for any bidding credit discount, then $D_{k,c} = 0$ for all assignment phase markets and categories.

If the bidder qualifies for the rural service provider bidding credit *or* if the bidder qualifies for the small business bidding credit and did not reach the small markets cap, then

$$D_{k,c} = \frac{GP_{k,c}}{\sum_{k'\in M}\sum_{c'\in C} GP_{k',c'}} \cdot TD$$

That is, the total discount is apportioned to assignment phase markets proportionally to the sum of the bidder's gross payment in each market and category.

For each assignment phase market, the calculation is rounded down to the nearest dollar. The slack due to rounding down is then distributed (one dollar at a time) to assignment phase markets and categories based on ascending order of the gross payments.

If the bidder qualifies for the small business bidding credit and it reached the small markets bidding credit cap, then

- If $k \in SM$,

$$D_{k,c} = \frac{GP_{k,c}}{\sum_{k' \in SM} \sum_{c' \in C} GP_{k',c'}} \cdot (\$10 \text{ million})$$

- If $k \notin SM$,

$$D_{k,c} = \frac{GP_{k,c}}{\sum_{k' \in M \setminus SM} \sum_{c' \in C} GP_{k',c'}} \cdot (TD - \$10 \text{ million})$$

That is, the \$10 million discount that applies to small markets is apportioned to assignment phase markets that consist of PEAs subject to the small market bidding cap proportionally to the bidder's gross payments for those assignment phase markets. The remaining discount (*i.e.*, $TD - \$10$ million) is apportioned among the assignment phase markets that consist of PEAs not subject to the small market bidding cap.

For each assignment phase market, the calculation is rounded down to the nearest dollar. The slack due to rounding down is then distributed (one dollar at a time) to assignment phase markets and categories based on ascending order of the gross payments.

8.2 Apportioning Assignment Phase Payments and Discounts to Licenses

Suppose that the bidder has been assigned a license in PEA j and category c. Let k be the assignment phase market in which PEA j was assigned.

The gross post-auction price of the license is determined by the following formula:

$$FCP_{j,c} + \frac{FCP_{j,c}}{\sum_{j' \in L_k} FCP_{j',c}} \cdot AP_{k,c}$$

That is, for each assignment phase market and category, the assignment phase payment is apportioned to the licenses in that assignment phase market and category in proportion to the final clock phase price of each license. Note that if the bidder's assignment payment in an assignment phase market is zero, then the gross post-auction price of each license it is assigned in that market is simply the final clock phase price for that license.

The net post-auction price of the license is determined by the following formula:

$$FCP_{j,c} + \frac{FCP_{j,c}}{\sum_{j' \in L_k} FCP_{j',c}} \cdot (AP_{k,c} - D_{k,c})$$

That is, for each assignment phase market, its assignment payment and its discount (see Section 8.1 for how the discount is determined) are apportioned to the licenses in that assignment phase market in proportion to the final clock phase price of each license. Note that if the bidder does not qualify for a bidding credit and its assignment payment in an assignment phase market and category is zero, then the net post-auction price of each license it is assigned in that market and category is simply the final clock phase price for that license.

Each license calculation is rounded down to the nearest dollar and then the slack due to rounding down is distributed to licenses (one dollar at a time) based on ascending order of the final clock phase prices.