Auction 103 Assignment Phase Technical Guide

1 Introduction

This technical guide sets forth the details of the bidding procedures for the assignment phase, the second phase of Auction 103 as described in the *Auction 103 Procedures Public Notice*.[1] The assignment phase offers winners of generic spectrum blocks in the clock phase of Auction 103 the opportunity to bid for preferred specific frequency assignments. Using the procedures adopted in the *Auctions 103 Procedures Public Notice* and detailed below, the bidding system will determine specific license assignments and any payment, above the final clock phase price, that a winning bidder will pay for the assignment. The bidding system will also determine specific license assignments for any 39 GHz incumbent that accepted modified licenses.[2]

The assignment phase is designed to promote two major goals. One goal is to make bidding relatively straightforward, although the underlying allocation problem is complex. The procedure promotes simplicity in several ways. First, to reduce the total number of bids that each bidder must make, it groups together non-top 20 PEAs within a region under certain conditions. Second, to simplify bidding strategy for bidders, it uses a *second-price* type of pricing rule that encourages bidders to bid according to their actual values for different frequencies while ensuring that there is no bidder or group of bidders willing to pay more for an alternative feasible assignment. Third, a winner does not need to make any additional bids – or make any additional payments – in the assignment phase. This makes bidding easier in both the assignment phase and the clock phase of the auction, because bidders in the clock phase will know that they need not pay more for licenses than the prices bid in the clock phase.

A second, equally important goal is to promote efficient and intensive use of the spectrum. To achieve that, the assignment phase rules ensure that each bidder is assigned contiguous frequencies within each category in each PEA, even if the bidder does not participate in the assignment phase. Each 39 GHz incumbent that accepted modified licenses will also be assigned contiguous frequencies in Category M/N.[3]

2 Assignment Rounds

The assignment phase consists of a series of *assignment rounds.* In each assignment round, licenses are assigned in up to six *assignment phase markets*, with each assignment phase market consisting of either a single PEA or a group of PEAs; see Section 2.1. Winning bidders from the clock phase that have a preference for specific license frequencies submit sealed bids for those licenses, separately for each category. Once an assignment round concludes, an optimization is solved for each category in each assignment phase market to assign specific frequency licenses to each winning bidder from the clock phase and to each 39 GHz incumbent that accepted modified licenses. Additional optimizations are

[1] *See Incentive Auction of Upper Microwave Flexible Use Service Licenses in the Upper 37 GHz, 39 GHz, and 47 GHz Bands for Next-Generation Wireless Services; Notice and Filing Requirements, Minimum Opening Bids, Upfront Payments, and Other Procedures for Auction 103; Bidding in Auction 103 Scheduled to Begin December 10, 2019*, Public Notice, FCC 19-63 (July 11, 2019) (*Auction 103 Procedures Public Notice*).

[2] Incumbents that elect to receive modified licenses will be assigned frequencies based on the results of the auction assignment phase but cannot bid for particular frequencies in the assignment phase. *See Reconfigured 39 GHz Incumbent Holdings; Initial Commitment Options and Timeline; Preparation for Incentive Auction of Upper Microwave Flexible Use Service Licenses in the Upper 37 GHz, 39 GHz, and 47 GHz Bands (Auction 103),* Public Notice, DA 19-503 at 3, para. 8 (WTB/OEA June 5, 2019).

[3] *Id.* at 3-4, para. 9.

solved to determine each bidder's assignment payment, which will be equal to or less than the bidder's bid value for the assignment.

The bidding system will determine whether to group PEAs into a single assignment phase market according to the rule detailed in Section 2.1 below.

2.1 Grouping PEAs into a Single Assignment Phase Market

A set of PEAs P will be grouped into one *assignment phase market* if all of the following four conditions are met:[4]

1. The PEAs in P are all in the same Regional Economic Area Grouping ("REAG")[5] and not in the top-20 PEAs;[6]
2. Either all PEAs in P are not subject to the small market bidding credit cap[7] or all PEAs in P are subject to the small market bidding credit cap;
3. For each PEA in P, the same entities (winning bidders and incumbents that accepted modified licenses) need to be assigned the same number of blocks in each category; and
4. For each PEA in P, the number of blocks that an incumbent accepting modified licenses cannot be assigned in the PEA due to a frequency assignment waiver must be the same.[8]

Because of this grouping of PEAs, the number of assignment phase markets will be smaller than or equal to the number of PEAs.

<u>Example 1</u>: REAG Grouping

PEA-060, PEA-069, and PEA-077 are all in REAG 1. These PEAs are not top-20 PEAs and are not subject to the small market bidding credit cap.

Suppose that one 39 GHz incumbent with licenses in these PEAs chose to configure its existing licenses and keep 2 blocks in each of these PEAs. All other 39 GHz incumbents in these PEAs chose to relinquish

[4] However, a PEA will not be grouped with other PEAs if (i) there are FCC-held blocks in Category M/N, and (ii) exactly one 39 GHz incumbent that accepted modified licenses kept a partial license in that PEA. Such a PEA is not grouped with other PEAs because, as described in Section 4, the FCC-held licenses will be contiguous to the partial license of the 39 GHz incumbent in that PEA.

[5] The six Regional Economic Area Groupings (REAG) are: Northeast, Southeast, Great Lakes, Mississippi Valley, Central, and West. Each of the remaining REAGs (i.e., Alaska, Hawaii, Puerto Rico and US Virgin Islands, Guam and the Northern Mariana Islands, American Samoa, and the Gulf of Mexico) will be merged in one of the 6 main REAGs.

[6] The top-20 PEAs are PEAs 1–20.

[7] PEAs that are subject to the small market bidding credit cap are those PEAs with a population of 500,000 or less, which corresponds to PEAs 118–416, excluding PEA 412. *See Updating Part 1 Competitive Bidding Rules et al.*, Report and Order, Order on Reconsideration of the First Report and Order, Third Order on Reconsideration of the Second Report and Order, Third Report and Order, 30 FCC Rcd 7493, 7546-47, paras. 127-28 (2015).

[8] Incumbents that do not bid in the auction and instead receive modified licenses may, under certain circumstances, be assigned frequencies not subject to site-specific Federal coordination, if they are granted a waiver under the process the Commission established in the *Auction 103 Procedures Public Notice*. *See Auction 103 Procedures Public Notice* at 53-54, paras. 162-63. Pursuant to section 30.205 of the Commission's rules, licensees operating in 37-38.6 GHz located within specified zones defined by the coordinates provided must coordinate their operations with the Department of Defense via the National Telecommunications and Information Administration (NTIA), and licensees operating only in 37-38.0 GHz must coordinate their operations with Federal Space Research Service (space to Earth) users of the band via the NTIA. *See* 47 CFR § 30.205.

all their licenses in the auction. Thus, the supply for Category M/N in the clock phase was 22 for each of these PEAs.

In each of these PEAs:

- Bidder #1 won 8 Category M/N blocks in the clock phase.
- Bidder #2 won 10 Category M/N blocks in the clock phase.
- Bidder #3 won 4 Category M/N blocks and 4 Category P blocks in the clock phase.
- Bidder #4 won 6 Category P blocks in the clock phase.

Then PEA-060, PEA-069, and PEA-077 will be grouped and treated as a single combined market for the assignment phase.

Example 2: Not Possible to Group

PEA-043 and PEA-045 are both in REAG 2. These PEAs are not in the top-20 PEAs and are not subject to the small market bidding credit cap. Suppose that all 39 GHz incumbents in each of these PEAs chose to relinquish all their licenses in the auction, so there are 24 blocks of Category M/N in each PEA.

The winners in these two PEAs are as follows:

In PEA-043:

- Bidder #1 won 10 Category M/N blocks in the clock phase.
- Bidder #2 won 10 Category M/N blocks in the clock phase.
- Bidder #3 won 4 Category M/N blocks and 4 Category P blocks in the clock phase.
- Bidder #4 won 6 Category P blocks in the clock phase.

In PEA-045:

- Bidder #1 won 10 Category M/N blocks in the clock phase.
- Bidder #2 won 10 Category M/N blocks in the clock phase.
- Bidder #3 won 4 Category M/N blocks and 4 Category P blocks in the clock phase.
- Bidder #5 won 6 Category P blocks in the clock phase.

Then, PEA-043 and PEA-045 will not be grouped, because they do not have identical clock phase winners. Specifically, Bidder #4 won 6 Category P in PEA-043 but did not win any blocks in PEA-045, whereas Bidder #5 won 6 Category P blocks in PEA-045 but did not win any blocks in PEA-043.

2.2 Sequencing of Assignment Rounds

The assignment phase begins with assignment rounds for top-20 PEAs (PEAs 1-20). The top-20 PEAs are ordered in descending order of pops, and bidding is conducted for a single assignment phase market per round, sequentially. Note that there is no grouping for the top-20 PEAs.

After bidding has been conducted for the top-20 PEAs, bidding is conducted simultaneously for the six REAGs, but in descending order of pops within each REAG.[9] That is, bidding may be conducted for up

[9] With the exception of PEA 412, Puerto Rico, the PEA numbering is in descending order of pops.

to six assignment phase markets at the same time, in order to speed up the assignment phase. The rounds continue until all assignment phase markets are assigned.

If an assignment phase market consists of multiple PEAs, its pops will be set to be equal to the sum of the pops of the PEAs that it comprises, for purposes of determining the sequencing.

Before bidding for the assignment phase starts, the bidding system will inform bidders about which PEAs have been grouped and the sequencing of assignment rounds.

The following tables show two examples of the sequencing of assignment phase markets. In the first example, there is no grouping, that is, each assignment phase market consists of a single PEA. In the second example, some assignment phase markets consist of multiple PEAs and, as a result, there are fewer assignment rounds.

Table 1: Sequencing of assignment phase markets with no grouping

Round	PEA					
1	001					
2	002					
…	…					
20	020					
	REAG 1	REAG 2	REAG 3	REAG 4	REAG 5	REAG 6
21	041	412	023	024	028	022
22	044	021	025	030	035	026
…	…	…	…	…	…	…

Table 2: Sequencing of assignment phase markets with grouping

Round	PEA(s)					
1	001					
2	002					
…	…					
20	020					
	REAG 1	REAG 2	REAG 3	REAG 4	REAG 5	REAG 6
21	041; 044	412	023	024	028	022; 034
22	048	021; 029; 033	025	030	035	026
…	…	…	…	…	…	…

As illustrated in the tables above, after bidding for the top-20 PEAs is finished, bidding for multiple assignment phase markets will be conducted in the same round.

3 Bidding

3.1 Bidding Options

For each assignment phase market and each category, the bidding system will determine all contiguous assignment options where the number of licenses per PEA is equal to the number of blocks that the winner has won in the clock phase in that category and assignment phase market.[10] This set is referred to as the *bidding options* of the bidder. Note that the bidding options of a bidder do not depend on the clock phase winnings of other bidders.

Example 1: The bidder won one Category M/N block for each PEA of a given assignment phase market. Then, the bidder has 24 bidding options: M1, M2, …, M10, N1, N2, …, N14.

Example 2: The bidder won four Category P blocks for each PEA of a given assignment phase market. Then, the bidder has the following 7 bidding options: P1-P4, P2-P5, P3-P6, P4-P7, P5-P8, P6-P9, and P7-P10.

Example 3: The bidder won four Category M/N blocks and four Category P blocks for each PEA of a given assignment phase market. Then, the bidder will have a set of bidding options for each category.

For Category M/N, the 21 bidding options are: M1-M4, M2-M5, M3-M6, M4-M7, M5-M8, M6-M9, M7-M10, M8-N1, M9-N2, M10-N3, N1-N4, N2-N5, N3-N6, N4-N7, N5-N8, N6-N9, N7-N10, N8-N11, N9-N12, N10-N13, N11-N14.

For Category P, the 7 bidding options are: P1-P4, P2-P5, P3-P6, P4-P7, P5-P8, P6-P9, and P7-P10.

The bidder can bid on any of its bidding options. However, note that, in certain instances, some of the bidding options cannot be assigned. For instance, if another bidder won six blocks for Category P, the bidder of Example 3 above cannot be assigned P2-P5, because, if it were, it would not be possible to assign contiguous spectrum to the other bidder. The bidding options of a bidder are not limited only to the options that can be won by the bidder, because limiting the bidding options in that way may permit a bidder to infer the clock phase winnings of other bidders.

3.2 Automatic Assignments and Pre-Assignments

If a bidder has only one bidding option available in an assignment phase market for a category, the bidder will be automatically assigned the licenses in this option and will not be able to submit bids for this assignment phase market. For example, a bidder that won ten Category P blocks in a PEA will be automatically assigned P1-P10.

If all winners in a PEA have only one bidding option available to each of them, then all licenses in that PEA are pre-assigned to winners and there will not be a round held for that PEA.

If a PEA has no winners from the clock phase (all blocks remained FCC-held), there will not be a round held for that PEA. If such a PEA has one or more incumbents that accepted modified licenses, the system will determine an assignment for each of these incumbents, and the incumbents will be informed about their assignments after the conclusion of the assignment phase.

3.3 Bidding Rules

A bidder may specify a bid value for each bidding option that it is presented for an assignment phase market. The bidder bids for a bidding option by specifying a non-negative whole dollar amount for that option.

[10] An incumbent accepting modified licenses may, under certain circumstances, be assigned frequencies not subject to site-specific Federal coordination, if it is granted a waiver under the process the Commission established in the *Auction 103 Procedures Public Notice. See supra* note 8. If an incumbent receives such a waiver, its bidding options will not include any licenses in frequencies where there are established Federal coordination zones as specified in the waiver order.

A bidder that does not have clock phase winnings in a PEA will not have any bidding options in the corresponding assignment phase market and thus cannot submit bids for that market.

If an assignment phase market is a group of PEAs, then each bidder has the same clock phase winnings in each of those PEAs (because of the grouping rule described in Section 2.1). By specifying a bid value for a bidding option, the bidder indicates the maximum amount that it is willing to pay to be assigned that option in all those PEAs. A bidder will not be able to bid for different frequency assignments in the various PEAs within a group.

A winner in the clock phase is not required to bid in the assignment phase. In particular, such a bidder may not wish to bid if it is indifferent among all its possible assignments. The bidding system will consider a bid value of zero for any bidding option for which a bidder submits no bid.

3.4 Winning Assignments and Payments

After bidding in an assignment round concludes, the bids are processed to determine the winning assignments and the payments for that round. For each assignment phase market of the round, each bidder is then informed about its winning assignment and its assignment payment for each category in that assignment phase market. This information is disclosed to the bidder before the next assignment round starts.

4 Assignment Determination

For a given category in a given assignment phase market, an assignment is *feasible* if:

(1) Each bidder is assigned one of its bidding options;

(2) Any FCC-held licenses are contiguous;

(3) In the case of Category M/N where one or more 39 GHz incumbents chose to accept modified licenses and hold a positive quantity in that PEA, each such incumbent is assigned contiguous licenses;

(4) In the case of Category M/N, if an incumbent accepting modified licenses has been granted a waiver relating to its frequency assignment in that PEA, the incumbent is not assigned any licenses in frequencies where there are established Federal coordination zones as specified in the waiver order;[11] and

(5) In the case of Category M/N, if (i) there are FCC-held blocks in the category, and (ii) exactly one 39 GHz incumbent accepted modified licenses and kept a partial license in that PEA, then the FCC-held licenses are contiguous to the partial license of that 39 GHz incumbent.[12] In this case, the FCC-held licenses will be assigned to frequencies immediately lower than the partial license.

The winning assignment for a given assignment phase market is determined separately for Category M/N and Category P by maximizing the sum of bid values across all bids for the category. Ties, if any, are broken by including pseudo-random numbers in an optimization.

[11] *See* 47 CFR § 30.205.

[12] If there are FCC-held blocks in the PEA and two or more 39 GHz incumbents accepted modified licenses and kept partial licenses in that PEA, there is no requirement for any partial licenses to be contiguous with the FCC-held licenses.

Specifically, the assignment determination is done by solving two optimization problems for each category in each assignment phase market. The first optimization problem finds the maximum sum of bid values among all feasible assignments. The bidding system then solves another optimization problem using randomly generated numbers to break ties, if any. The solution to the latter optimization is selected as the final assignment.

To mathematically formulate the assignment determination, the following notation is used:

- B denotes the set of bidders in that assignment phase market and category, that is, the set of winners in this particular assignment phase market and this particular category from the clock phase.
- *Relevant only for Category M/N*: I denotes the set of incumbents that accepted modified licenses in the PEA.
- The FCC is referred to as bidder 0.
- $B \cup I \cup \{0\}$ is used to denote the set of bidders, incumbents and the FCC.
- *K* denotes the set of licenses for the category. When the category is M/N, *K* consists of licenses M1, M2, …, M10, N1, N2, …, N14. When the category is P, *K* consists of licenses P1, P2, …, P10.
- S denotes a set of licenses. For each license j, S_j denotes the indicator variable of whether license j is in set S. That is, $S_j = 1$ if $j \in S$, and $S_j = 0$ if $j \notin S$.
- m_i is the number of blocks per PEA won by bidder i or held by incumbent i after reconfiguration (applicable only to Category M/N) in the assignment phase market and category of interest.
- F_i denotes the set of feasible assignments for bidder or incumbent i. That is, F_i will consist of all sets $S \subseteq K$ with m_i contiguous licenses. For bidder i, F_i is the set of its bidding options in the category and assignment phase market (see Section 3 for examples). For an incumbent i that has been granted a waiver, F_i will only include assignments for which all licenses are not within relevant established Federal coordination zones.[13]
- m_0 is the number of FCC-held blocks per PEA in the assignment phase market and category of interest.
- F_0 consists of all sets $S \subseteq K$ with m_0 contiguous licenses. This set gives the possible assignments for any FCC-held licenses.
- $b_i(S)$ denotes the bid value of bidder i for set $S \in F_i$. If the bidder did not bid on one or more of its bidding options, then the value of each of these bids is zero.
- b denotes the set of bid values.

In the case of Category M/N, if (i) there are FCC-held blocks in the PEA, and (ii) exactly one incumbent that accepted modified licenses kept a partial license in the PEA, then the FCC-held licenses need to be contiguous to the partial license of that 39 GHz incumbent. This is achieved by treating the FCC and the incumbent as one entity for the purposes of the assignment determination. Specifically:

- "Bidder 0" will refer to the FCC together with the incumbent that kept a partial block in the PEA;

[13] *See* 47 CFR § 30.205.

- m_0 will equal the number of FCC-held blocks plus the number of blocks held by that incumbent after reconfiguration (including the partial block);
- The set F_0 will include all possible joint assignments for the FCC-held licenses and that incumbent;
- The incumbent with the partial block will not be included in the set I; and
- Once the assignment is determined, the lowest frequency licenses in the assignment will be assigned as FCC-held.

Variable Definitions:

$X_i(S)$ is a binary decision variable which has value 1 if exactly the licenses of set S are assigned to bidder i or incumbent i, and 0 otherwise. This variable is defined for all $i \in B \cup I \cup \{0\}$. Thus, $X_0(S) = 1$ if the set of licenses assigned to the FCC is S.

4.1 Maximum Sum of Bid Values

$$r(b) = max \sum_{i \in \mathrm{B}} \sum_{S \in F_i} b_i(S) \cdot X_i(S)$$

Subject to:

$\sum_{i \in B \cup I \cup \{0\}} \sum_{S \in F_i} S_j \cdot X_i(S) = 1$	$\forall j \in K$	(1)
$\sum_{S \in F_i} X_i(S) = 1$	$\forall i \in B \cup I \cup \{0\}$	(2)
$X_i(S) \in \{0,1\}$	$\forall i \in B \cup I \cup \{0\}, \forall S \in F_i$	(3)

The objective function is equal to the sum of bid values of an assignment, across all bidders.

Explanation of Constraints:

- Constraint (1) ensures that each license is assigned exactly once, either to one of the bidders or to one of the incumbents or to the FCC.
- Constraint (2) ensures that each bidder is assigned exactly one of its bidding options, that each incumbent is assigned contiguous licenses, and that the set of licenses assigned to the FCC is contiguous.
- Constraint (3) states that each decision variable $X_i(S)$ can be either equal to 0 or 1.

4.2 Tie-breaking

For every set S and every bidder $i \in B$ and every incumbent $i \in I$, the bidding system generates a pseudo-random number $\xi_i(S)$ drawn uniformly at random from the set $\{1,2, \dots, 10^8\}$. The bidding system then solves an optimization problem to find the assignment that maximizes the sum of pseudorandom numbers among all assignments that satisfy constraints (1) through (3) of Section 4.1 such that the sum of bid values is equal to $r(b)$. In particular, the objective of the optimization problem is:

$$max \sum_{i \in \mathrm{B} \cup I} \sum_{S \in F_i} \xi_i(S) \cdot X_i(S)$$

In addition to constraints (1) – (3), the tie-breaking optimization has the following constraint:

$$\sum_{i \in B} \sum_{S \in F_i} b_i(S) \cdot X_i(S) \geq r(b) \tag{4}$$

This constraint states that the sum of bid values must be greater than or equal to the result of the optimization of Section 4.1.

5 Assignment Payment Determination

A bidder's assignment payment for the set of licenses it is assigned for a category in an assignment phase market is an additional payment amount above its gross final clock phase payments. If a bidder did not bid (or submitted a bid of zero) for the set of licenses that it is assigned, then no additional calculation is necessary, and the bidder will not have any additional assignment payment for that category in that assignment phase market. If, on the other hand, the bidder submitted a positive bid for the winning assignment, then the bidding system will calculate a type of 'second-price' assignment payment.[14]

The bidding system will apply bidder-optimal core prices and will use the "nearest Vickrey" approach to determine the assignment payments. In some cases, the second price (Vickrey price) may not be high enough to ensure that no bidder or group of bidders is willing to pay more for an alternative feasible assignment, and so an additional payment above Vickrey prices may be required. In the event that such a payment is required, the calculation of the additional payment to be paid by each bidder will be weighted based on the number of blocks won by the bidder in the clock phase for that category in that assignment phase market.

The assignment payments will satisfy the following conditions:

First condition: Each assignment payment must be positive or zero and not more than the dollar amount of the winning assignment phase bid.

Second condition: The set of assignment payments must be sufficiently high that there is no bidder or group of bidders willing to pay more for an alternative feasible assignment. If there is only one set of assignment payments that satisfies the first two conditions, this determines the assignment payments.

Third condition: If there are many sets of assignment payments that fulfil the first and second conditions, the set(s) of assignment payments minimizing the sum of assignment payments across all bidders is (are) selected. If there is only one set of assignment payments that satisfies these three conditions, this determines the assignment payments.

Fourth condition: If there are many sets of assignment payments that satisfy the first three conditions, the set of assignment payments that minimizes the weighted sum of squares of differences between the assignment payments and the Vickrey prices will be selected. The weighting is relative to the number of blocks won by the bidder in the category in the assignment phase market. This approach for selecting among sets of assignment payments that minimize the sum of assignment payments across bidders is referred to as the "nearest Vickrey" approach.

Section 5.1 describes how the Vickrey prices are calculated. Section 5.2 describes how payments are adjusted (if needed) to ensure that there is no bidder or group of bidders willing to pay more for an alternative feasible assignment. Section 5.3 provides an example.

5.1 Vickrey Price Calculation

For each bidder, the bidding system will determine the Vickrey price by re-solving the optimization problem of Section 4.1, but setting all bids of the bidder to zero while keeping the bids of every other bidder unchanged from the prior optimization, and calculate a hypothetical maximum sum of bid values

[14] In some cases, this may also be zero.

from that optimization. The difference between the maximum sum of bid values associated with the actual optimization and the hypothetical maximum sum of bid values that would occur had that bidder provided all bids of zero will indicate the amount by which the bidder's winning bid amount exceeded the minimum amount it would have needed to bid to ensure the same winning assignment set. The Vickrey price is calculated by subtracting that amount from the bidder's actual bid amount.

Specifically: Let $r(b)$ denote the maximum value attained by solving the optimization problem of Section 4.1 when the set of bid values is b. Let b_i^* be the bid amount of bidder i for the bidding option that it is assigned.

The Vickrey price of bidder i for a given category in a given assignment phase market is:

$$p_i^{Vickrey} = b_i^* - (\, r(b) - r(b_{i\to 0})\,)$$

where $b_{i\to 0}$ represents the set of bid values where the bid values of all bids of bidder i are set to zero and the bid values of every other bidder are not changed.

5.2 Core Adjustments

An extra payment beyond the Vickrey prices is sometimes required in order to satisfy the second condition, which requires that the set of assignment payments is sufficiently high that there is no bidder or group of bidders prepared to pay more for an alternative feasible assignment. A bidder or group of bidders willing to pay more for an alternative feasible assignment is referred to as a blocking coalition of bidders. The group that is willing to pay the most forms the first blocking coalition. A blocking coalition is unblocked by increasing the assignment payments while ensuring that assignment payments are increased no more than necessary and that each bidder's assignment payment is less than or equal to the corresponding bid amount. After adjusting the assignment payments to unblock the first blocking coalition, additional blocking coalitions may exist which are each unblocked by again increasing the assignment payments until there is no bidder or group of bidders willing to pay more for an alternative feasible assignment. Each bidder's assignment payment is guaranteed to be at least its Vickrey price and no more than its bid amount for its assignment.

Assignment payments can be calculated iteratively via a core adjustment process. This process operates by starting with Vickrey prices and then by iteratively adjusting assignment payments until there is no bidder or group of bidders willing to pay more for an alternative feasible assignment. It does so by gathering pricing constraints from each blocking coalition and then satisfying the pricing constraints by selecting assignment payments which minimize the distance, weighted by the number of blocks won by each bidder in the category in the given assignment phase market, from Vickrey prices.

To mathematically formulate the core adjustment calculations, the following notation is used in addition to the notation of Section 4:

- b_i^* is the bid amount of bidder i for the bidding option that it is assigned (in the original assignment determination problem).
- p_i^n is the payment of bidder i in iteration n.
- C^n denotes the blocking coalition for iteration n.

In the first iteration, the payment of bidder i is set equal to its Vickrey price, that is, $p_i^1 = p_i^{Vickrey}$.

Given a set of assignment payments for iteration n, calculate a reduced bid for each bidder and each of its bidding options by deducting the current surplus of the bidder $(b_i^* - p_i^n)$ from the corresponding bid amount. Specifically, in iteration n, the reduced bid of bidder i for a bidding option $S \in F_i$ is:

$$b_i^n(S) = \max\{b_i(S) - (b_i^* - p_i^n), 0\}$$

Let C^n be the set of bidders with strictly positive bid amounts for the options they are assigned in the solution to the assignment determination problem for the set of reduced bids b^n in the optimization problem of Section 4.1. These bidders form the potential blocking coalition for iteration n. Among all potential blocking coalitions for iteration n, C^n is the one with the highest value, that is, $r(b^n)$.

If the value of the potential blocking coalition for iteration n does not exceed the sum of assignment payments for iteration n (that is, if $r(b^n) \leq \sum_{i \in B} p_i^n$), then there is no blocking coalition for iteration n and p_i^n represents the assignment payment for bidder i. In this case, the assignment payment for bidder i is determined by rounding p_i^n up to the nearest integer, and no further calculations are required.

Otherwise, bidders in C^n do form a blocking coalition for iteration n, and the bidding system will calculate the updated set of assignment payments p_i^{n+1} as described below.

The bidding system will first calculate the minimum sum of assignment payments required to unblock all coalitions as of iteration n. This is done by solving the following optimization problem:

Minimize Sum of Assignment Payments (for third condition)

$$\mu^n = \min \sum_{i \in B} p_i$$

Subject to:

$\sum_{i \in B \setminus C^k} p_i \geq r\left(b^k\right) - \sum_{i \in C^k} p_i^k$	$\forall k \in \{1, \dots, n\}$	(1)
$p_i \geq p_i^{Vickrey}$	$\forall i \in B$	(2)
$p_i \leq b_i^*$	$\forall i \in B$	(3)

Explanation of Constraints:

- Constraint (1) ensures that all coalitions are unblocked, that is, for the blocking coalition of iteration k, the sum of assignment payments by bidders not in the coalition must be greater than or equal to the value of the coalition under the set of reduced bids for iteration k less the total iteration k payments of bidders in that coalition. There is one such constraint for each iteration.
- Constraint (2) requires that the price paid by each bidder be greater than or equal to the bidder's Vickrey price.
- Constraint (3) requires that the price paid by each bidder be less than or equal to the bidder's bid amount for its winning assignment.

The bidding system will then update the assignment payments of bidders such that they sum up to μ^n (the third condition).

If there is more than one set of assignment payments that sum up to μ^n, the set of assignment payments that minimizes the weighted sum of squares of differences between the assignment payments and the Vickrey prices will be selected. The weighting is relative to the number of blocks that the bidder won in that category in that assignment phase market (the fourth condition).

The updated assignment payments p_i^{n+1} are calculated as the optimal solution to:

Minimize Distance Between Assignment Payments and Vickrey Prices (for fourth condition)

$$\min \sum_{i \in B} \frac{(p_i^{n+1} - p_i^{Vickrey})^2}{m_i}$$

Subject to:

$\sum_{i \in B \setminus C^k} p_i^{n+1} \geq r(b^k) - \sum_{i \in C^k} p_i^k$	$\forall k \in \{1, \ldots, n\}$	(1)
$p_i^{n+1} \geq p_i^{Vickrey}$	$\forall i \in B$	(2)
$p_i^{n+1} \leq b_i^*$	$\forall i \in B$	(3)
$\sum_{i \in B} p_i^{n+1} = \mu^n$		(4)

This quadratic problem minimizes the weighted sum of squares of differences between the updated assignment payments p_i^{n+1} and the Vickrey prices $p_i^{Vickrey}$, weighted by the number of blocks (m_i) won by each bidder.

Constraints (1) through (3) are the same as in the previous optimization.

Explanation of New Constraint:

Constraint (4) ensures that the sum of the updated payments is equal to the minimum amount required to unblock all of the coalitions up to iteration n.

5.3 Assignment Payment Calculation Example

This section provides an example to illustrate how Vickrey prices and core adjustments are calculated in order to determine the assignment payments. In this example, Bidder 1 won two Category P blocks, and Bidders 2 and 3 won four Category P blocks each.

The bids of each bidder are shown in Table 3. The positive bid amounts are summarized below:

- Bidder 1 bids $1,000 on P9P10.
- Bidder 2 bids $2,000 on P3-P6.
- Bidder 3 bids $3,000 on P7-P10.

Assignment Determination: The sum of bid values is maximized when Bidder 1 is assigned P1P2, Bidder 2 is assigned P3-P6, and Bidder 3 is assigned P7-P10. The value of this assignment is equal to $5,000.

Vickrey Prices: In this example, all Vickrey prices are $0 even though Bidder 3 is assigned blocks for which Bidder 1 bid $1,000.

- The Vickrey price of Bidder 1 is equal to $0, because its bid value for its assignment is $0.
- To calculate the Vickrey price of Bidder 2, the bidding system would solve the assignment determination optimization problem with all of the bids of Bidder 2 set to $0. The optimal value of this optimization problem is $3,000, because Bidder 3 would be assigned P7-P10 (with a value of $3,000) and the remaining blocks would be assigned to Bidders 1 and 2 (with a value of $0). Thus, the Vickrey discount for Bidder 2 is equal to $5,000-$3,000 = $2,000. The Vickrey price of Bidder 2 is then calculated as its bid amount for its assignment ($2,000) less its Vickrey discount ($2,000) and is therefore equal to $0.
- To calculate the Vickrey price of Bidder 3, the bidding system would solve the assignment determination optimization problem with all of the bids of Bidder 3 set to $0. The optimal value of this optimization problem is $2,000, because Bidder 1 would be assigned P1P2 (with a value of $0), Bidder 2 would be assigned P3-P6 (with a value of $2,000) and Bidder 3 would be assigned blocks P7-P10 (with a value of $0). Note that it is not feasible to assign P9P10 to Bidder 1 and

P3-P6 to Bidder 2, because then it would not be possible to assign contiguous spectrum to Bidder 3. Thus, the Vickrey discount for Bidder 3 is equal to $5,000-$2,000 = $3,000. The Vickrey price of Bidder 3 is then calculated as its bid amount for its assignment ($3,000) less its Vickrey discount ($3,000) and is therefore equal to $0.

Iteration 1: The next step is to determine whether there is a blocking coalition for iteration 1. To do this, the bidding system would first calculate the surplus of each bidder for the first iteration as the bidder's bid amount for its assignment less its Vickrey price. The surplus is $0 for Bidder 1, $2,000 for Bidder 2, and $3,000 for Bidder 3. The bidding system would then calculate a set of reduced bids for each bidder by subtracting the bidder's surplus from its actual bid amount for each bidding option. If the result for a bidding option is negative, the reduced bid amount is set to be equal to $0. The reduced bid amounts for the first iteration, b^1, are shown in Table 3. The bids of Bidder 1 are not reduced because Bidder 1 derives $0 surplus. The bids of Bidder 2 are reduced by up to $2,000 while ensuring that all bid values are non-negative. Similarly, the bids of Bidder 3 are reduced by up to $3,000 while ensuring that all bid values are non-negative.

The assignment determination problem is solved with the set of reduced bids for iteration 1. There is a blocking coalition (consisting of Bidder 1) with value $1,000. That is, Bidder 1 would be willing to pay up to $1,000 more than the sum of Vickrey prices (which is $0) in order to be assigned P9P10. The bidding system then calculates that the minimum sum of assignment payments to unblock this coalition is $\mu^1 = 1{,}000$. Because in this example Bidders 2 and 3 won the same number of blocks, the assignment payment of each of those bidders will be incremented by the same amount, namely, by $500. Thus, the assignment payments for iteration 2 are: $p_1^2 = 0$ and $p_2^2 = p_3^2 = 500$.

Iteration 2: The bidding system checks whether there exists another blocking coalition by solving the assignment determination problem for the reduced set of bids b^2 shown in Table 3. The maximum sum of bids is equal to $1,000 which does not exceed the sum of assignment payments for this iteration. Thus, there does not exist a blocking coalition in iteration 2. This implies that the assignment payments are equal to p^2, that is, $0 for Bidder 1, $500 for Bidder 2 and $500 for Bidder 3.

Table 3: Assignment Payment Calculation Example

Bidders	*Bidder 1 (2 blocks)*	*Bidder 2 (2 blocks)*	*Bidder 3 (2 blocks)*
Bids	**P1P2: $0**	P1-P4: $0	P1-P4: $0
	P2P3: $0	P2-P5: $0	P2-P5: $0
	P3P4: $0	**P3-P6: $2,000**	P3-P6: $0
	P4P5: $0	P4-P7: $0	P4-P7: $0
	P5P6: $0	P5-P8: $0	P5-P8: $0
	P6P7: $0	P6-P9: $0	P6-P9: $0
	P7P8: $0	P7-P10: $0	**P7-P10: $3,000**
	P8P9: $0		
	P9P10: $1,000		
Vickrey prices (p^1)	$0	$0	$0
Bidder Surplus at p^1	$0	$2,000	$3,000
Reduced Bids Iteration 1 (b^1)	P1P2: $0	P1-P4: $0	**P1-P4: $0**
	P2P3: $0	P2-P5: $0	P2-P5: $0
	P3P4: $0	P3-P6: $0	P3-P6: $0
Coalition	P4P5: $0	P4-P7: $0	P4-P7: $0
$C^1 = \{1\}$	P5P6: $0	**P5-P8: $0**	P5-P8: $0
	P6P7: $0	P6-P9: $0	P6-P9: $0
	P7P8: $0	P7-P10: $0	P7-P10: $0
	P8P9: $0		
	P9P10: $1,000		
Adjusted payments p^2 (Assignment payments)	$0	$500	$500
Bidder Surplus at p^2	$0	$1,500	$2,500
Reduced Bids Iteration 2 (b^2)	**P1P2: $0**	P1-P4: $0	P1-P4: $0
	P2P3: $0	P2-P5: $0	P2-P5: $0
	P3P4: $0	**P3-P6: $500**	P3-P6: $0
No blocking	P4P5: $0	P4-P7: $0	P4-P7: $0
coalition	P5P6: $0	P5-P8: $0	P5-P8: $0
	P6P7: $0	P6-P9: $0	P6-P9: $0
	P7P8: $0	P7-P10: $0	**P7-P10: $500**
	P8P9: $0		
	P9P10: $1,000		

6 Final Auction Payment

This section describes how a bidder's final auction payment is calculated at the conclusion of the assignment phase.

The section uses the following notation:

- $d_{T,i,\{c,j\}}$ denotes the processed demand of bidder i for category c in PEA j after the final clock round. This is the number of blocks that the bidder won in category c in PEA j.
- $p_{T,\{c,j\}}$ denotes the posted price for category c in PEA j after the final clock round. This is the final clock phase price.
- R denotes the set of all products in the clock phase.
- $AP_{i,k,c}$ denotes the assignment payment of bidder i for category c in assignment phase market k.
- AM denotes the set of assignment phase markets.
- SM denotes the set of assignment phase markets that consist of PEAs that are small markets.[15]
- $APM(k)$ denotes the set of PEAs in assignment phase market k.
- w_j denotes the weighted MHz-pops of PEA j per block.
- $v_{i,j}$ denotes the relinquished weighted MHz-pops of bidder i for PEA j (in the 39 GHz band) after Round Zero.
- BC_i denotes the bidding credit percentage of bidder i.
- IP_i is the incentive payment of bidder i (applicable only to 39 GHz incumbents).
- $IP_{i,SM}$ is the part of the incentive payment of bidder i that relates to PEAs subject to the small market bidding credit cap (applicable only to 39 GHz incumbents).
- $IP_{i,NSM}$ is the part of the incentive payment of bidder i that relates to PEAs not subject to the small market bidding credit cap (applicable only to 39 GHz incumbents).
- FGP_i is the final gross payment of bidder i.
- FD_i is the final discount of bidder i.
- $GP_{i,k,c}$ denotes the gross payment of bidder i for category c in assignment phase market k. This is equal to the sum of the final clock phase prices for all licenses in the bidder's assignment *and* the bidder's assignment payment, for category c in assignment phase market k, that is,

$$GP_{i,k,c} = \sum_{j \in APM(k)} d_{T,i,\{c,j\}} \cdot p_{T,\{c,j\}} + AP_{i,k,c}$$

Moreover, the notation x^+ is used to denote the maximum between x and 0. That is, $x^+ = \max(x, 0)$.

When all assignment rounds have completed, a bidder's final gross payment is determined by summing the final clock phase prices across all licenses that it won and its assignment payments across all assignment phase markets. Equivalently, the final gross payment of bidder i is equal to the sum of its gross payments across all assignment phase markets and categories:

$$FGP_i = \sum_{k \in AM} \sum_{c \in \{MN,P\}} GP_{i,k,c}$$

[15] Note that according to the rules for grouping PEAs into a single assignment phase market described in Section 2.1, *either* all PEAs in an assignment phase market are subject to the small market bidding credit cap *or* none of the PEAs in that assignment phase market are subject to the small markets bidding credit cap.

For a bidder that does not qualify for a bidding credit and is not a relinquishing incumbent, the final auction payment is equal to its final gross payment.

For a bidder that qualifies for a bidding credit and/or is a relinquishing incumbent, the *final auction payment* is equal to its final gross payment *minus* its incentive payment (applicable to relinquishing incumbents) *minus* its final discount (applicable to bidders that qualify for a bidding credit), that is, $FGP_i - IP_i - FD_i$.

An incumbent's incentive payment is determined after the final clock round. If bidder i is a relinquishing incumbent, its incentive payment is equal to:

$$IP_i = \sum_{j=1}^{416} \frac{v_{i,j}}{w_j} \cdot p_{T,\{MN,j\}}$$

The system will first calculate the summation shown above and then round the result to the nearest dollar.

Note that an incumbent's incentive payment is determined at the conclusion of the clock phase and is based on the final clock phase prices. An incumbent does not receive any additional incentive payment in the assignment phase.

The remainder of this section describes how a bidder's final discount is calculated.

If bidder i qualifies for the rural service provider bidding credit and is not a 39 GHz incumbent, then its final discount is:

$$FD_i = \min\{\$10 \text{ million}, BC_i \cdot FGP_i\}$$

If bidder i qualifies for the rural service provider bidding credit and is a 39 GHz incumbent, then its final discount is:

$$FD_i = \min\{\$10 \text{ million}, BC_i \cdot (FGP_i - IP_i)^+\}$$

That is, the bidder gets a discount only if its final gross payment exceeds its incentive payment.

If bidder i qualifies for the small business bidding credit and is not a 39 GHz incumbent, then its final discount is:

$$FD_i = \min\left\{\$25 \text{ million}, BC_i \cdot \sum_{k \in AM \setminus SM} \sum_{c \in \{MN,P\}} GP_{i,k,c} + \min\left\{\$10 \text{ million}, BC_i \cdot \sum_{k \in SM} \sum_{c \in \{MN,P\}} GP_{i,k,c}\right\}\right\}$$

If bidder i qualifies for the small business bidding credit and is a 39 GHz incumbent, then its final discount FD_i can be calculated as the minimum of FD_i^1 and FD_i^2, where:

i.

$$FD_i^1 = BC_i \cdot (FGP_i - IP_i)^+$$

ii.

$$FD_i^2 = \min\left\{\$25\text{ million}, BC_i \cdot \left(\sum_{k\in AM\setminus SM}\sum_{c\in\{MN,P\}} GP_{i,k,c} - IP_{i,NSM}\right)^+ + \min\left\{\$10\text{ million}, BC_i \cdot \left(\sum_{k\in SM}\sum_{c\in\{MN,P\}} GP_{i,k,c} - IP_{i,SM}\right)^+\right\}\right\}$$

The first quantity (FD_i^1) considers the bidder's final gross payment and its incentive payment across all markets together and does not take into account any caps. The second quantity (FD_i^2) considers the bidder's discount in small markets and non-small markets separately. This calculation first caps the bidder's discount in small markets at $10 million, then adds the bidder's discount from all other markets (*i.e.*, markets that are not subject to the small market bidding cap) and caps the sum at $25 million.

All the discount calculations described above will be rounded to the nearest dollar. Rounding will only be done at the very end of a given calculation, that is, after all summations, multiplications, and minimization in a formula have been performed.

7 Calculations for Payment Information During the Assignment Phase

While winning bidders will be expected to pay the final auction payment set forth immediately above, the bidding system will show each bidder a running estimate of its auction payment obligations during the assignment phase. After each assignment round, each bidder will be shown its current gross total payment. A bidder that qualifies for a bidding credit will also be shown the net total payment as well as the corresponding capped and uncapped discounts. This way a bidder will know if it has reached any applicable bidding credit caps and the amount by which it is under or over.

7.1 Gross Total Payment

The *gross total payment* of bidder i when A is the set of assignment phase markets for which an assignment has been processed is calculated as:

$$GTP_i(A) = \sum_{r\in R} d_{T,i,r} \cdot p_{T,r} + \sum_{k\in A}(AP_{i,k,MN} + AP_{i,k,P})$$

The first term, which represents the clock phase payment, is the product of the final clock phase price and the number of blocks won by the bidder, summed over all products. The second term is the sum of the bidder's assignment payments across all assignment phase markets that have been assigned so far. When an assignment has been processed for all assignment phase markets, the bidder's gross total payment is equal to its final gross payment.

7.2 Bidding Credit Discounts

This section provides formulas for a bidder's capped and uncapped total payment discounts when A is the set of assignment phase markets for which an assignment has been processed. Note that A increases after every round.

A bidder's final discount is equal to its gross total payment discount when an assignment has been processed for all assignment phase markets.

All the discount calculations described in this section will be rounded to the nearest dollar. Rounding will only be done at the very end of a given calculation, that is, after all summations, multiplications, and minimization in a formula have been performed.

7.2.1 Non-Incumbent with Rural Service Provider Bidding Credit

In addition to its gross total payment, a bidder that qualifies for the rural service provider bidding credit is shown the corresponding uncapped discount, capped discount, and net total payment, after each assignment round.

Suppose that bidder i qualifies for the rural service provider bidding credit and is not an incumbent in the 39 GHz band, and A is the set of assignment phase markets for which an assignment has been processed.

The *uncapped total payment discount* of bidder i is calculated as:

$$BC_i \cdot GTP_i(A)$$

The *capped total payment discount* of bidder i is calculated as:

$$min\{\$10 \text{ million}, BC_i \cdot GTP_i(A)\}$$

That is, the bidder's gross total payment is multiplied by its bidding credit percentage and capped at $10 million.

7.2.2 Incumbent with Rural Service Provider Bidding Credit

Suppose that bidder i is an incumbent in the 39 GHz band who qualifies for the rural service provider bidding credit, and A is the set of assignment phase markets for which an assignment has been processed.

The *uncapped total payment discount* of bidder i is calculated as:

$$BC_i \cdot (GTP_i(A) - IP_i)^+$$

The *capped total payment discount* of bidder i is equal to the minimum of $10 million and the bidder's uncapped total payment discount:

$$min\{\$10 \text{ million}, BC_i \cdot (GTP_i(A) - IP_i)^+\}$$

That is, the incumbent receives a discount only on the difference between its gross total payment and its incentive payment, and the overall discount is capped at $10 million.

7.2.3 Non-Incumbent with Small Business Bidding Credit

In addition to its gross total payment, a bidder that qualifies for the small business bidding credit is shown the corresponding uncapped discount in small markets, uncapped discount, capped discount, and net total payment.

In this section, $GTP_{i,SM}(A)$ is used to denote the part of the gross total payment of bidder i that relates to PEAs subject to the small market bidding credit cap and $GTP_{i,NSM}(A)$ is used to denote the part of the gross total payment that relates to PEAs not subject to the small market bidding credit cap.

Suppose that bidder i qualifies for the small business bidding credit and is not an incumbent in the 39 GHz band, and A is the set of assignment phase markets for which an assignment has been processed.

The *uncapped total payment discount for small markets* only is:

$$BC_i \cdot GTP_{i,SM}(\mathrm{A})$$

The *uncapped total payment discount* is:

$$BC_i \cdot GTP_i(A)$$

The *capped total payment discount* is:

$$min\{\$25 \text{ million}, \ BC_i \cdot GTP_{i,NSM}(A) + min\{\$10 \text{ million}, BC_i \cdot GTP_{i,SM}(A)\}\}$$

This calculation first caps the discount from small markets at $10 million, then adds the discount from all other markets and caps the total at $25 million.

7.2.4 Incumbent with Small Business Bidding Credit

Suppose that bidder i is an incumbent in the 39 GHz band who qualifies for the small business bidding credit, and A is the set of assignment phase markets for which an assignment has been processed.

The *uncapped total payment discount for small markets* of bidder i is:

$$BC_i \cdot \left(GTP_{i,SM}(A) - IP_{i,SM}\right)^+$$

The *uncapped total payment discount* of bidder i is:

$$BC_i \cdot (GTP_i(A) - IP_i)^+$$

The *capped total payment discount* of bidder i is equal to the minimum of:

i.

$$BC_i \cdot (GTP_i(A) - IP_i)^+; \text{ and}$$

ii.

$$min\left\{\$25 \text{ million}, BC_i \cdot \left(GTP_{i,NSM}(A) - IP_{i,NSM}\right)^+ + min\left\{\$10 \text{ million}, BC_i \cdot \ \left(GTP_{i,SM}(A) - IP_{i,SM}\right)^+\right\}\right\}$$

7.3 Net Total Payment

The *net total payment* is equal to the bidder's gross total payment *minus* its incentive payment (applicable only for relinquishing incumbents) *minus* its capped total payment discount. Once all assignment rounds have been processed, a bidder's net total payment is equal to its final (net) auction payment.

A bidder's net total payment will be negative if the bidder's incentive payment for its relinquished licenses exceeds its obligation for new licenses.

8 Per-License Price Calculations

While final auction payments for winning bidders will be calculated as in Section 6 above, with bidding credit caps and assignment payments applying on an aggregate basis, rather than for individual licenses, the bidding system will also calculate a per-license price for each license. Such individual prices may be needed in the event that a licensee subsequently incurs license-specific obligations, such as unjust enrichment payments.

After the assignment phase, the bidding system will determine a net and gross price for each license that was won by a bidder by apportioning assignment payments and bidding credit discounts (only applicable for the net price) across all the licenses that the bidder won. To calculate the gross per-license price for a given license, the bidding system will apportion the assignment payment for the corresponding assignment phase market to the licenses that the bidder is assigned in that category and market in proportion to the final clock phase prices of those licenses. To calculate the net price, the bidding system will first apportion any applicable bidding credit discounts to each category and assignment phase market in proportion to the gross payment for that category and that market. Then, for each assignment phase market, the bidding system will apportion the assignment payment and the discount to licenses in proportion to the final clock phase prices of the licenses that the bidder is assigned in that category for that market.

This section uses the same notation as Section 6.

8.1 Apportioning Discounts to Each Category in Each Assignment Phase Market

This section describes how to apportion the bidder's final bidding credit discount across assignment phase markets and categories. Section 6 describes how the bidder's final discount (FD_i) is calculated. Let $D_{i,k,c}$ denote the discount that is apportioned to assignment phase market k and category c.

If bidder i does not qualify for any bidding credit discount or if bidder i is an incumbent whose incentive payment exceeds its final gross payment (and thus $FD_i = 0$), then $D_{i,k,c} = 0$ for all assignment phase markets and categories.

A bidder i that qualifies for the small business bidding credit and is not a 39 GHz incumbent is considered to have reached the small markets bidding credit cap if $BC_i \cdot \sum_{k\in SM}\sum_{c\in\{MN,P\}} GP_{i,k,c}$ rounded to the nearest integer is greater than or equal to $10 million.

Suppose that bidder i qualifies for the small business bidding credit and is a 39 GHz incumbent. Then its final discount is equal to the minimum of FD_i^1 and FD_i^2 (defined in Section 6). The bidder is considered to have reached the small markets bidding credit cap if $FD_i^1 > FD_i^2$ and $BC_i \cdot \left(\sum_{k\in SM}\sum_{c\in\{MN,P\}} GP_{i,k,c} - IP_{i,SM}\right)^+$ rounded to the nearest integer is greater than or equal to $10 million.

If bidder i qualifies for the rural service provider bidding credit *or* if the bidder qualifies for the small business bidding credit and did not reach the small markets cap, then

$$D_{i,k,c} = \frac{GP_{i,k,c}}{FGP_i} \cdot FD_i$$

That is, the final discount is apportioned to assignment phase markets proportionally to the bidder's gross payment in each market and category. Recall that the final gross payment, FGP_i, is equal to the sum of gross payments across all assignment phase markets and categories.

For each assignment phase market and category, the calculation is rounded down to the nearest dollar. The slack due to rounding down is then distributed (one dollar at a time) to assignment phase markets and categories based on ascending order of gross payments. Ties are broken based on ascending lexicographic order of assignment phase market and category ID. The assignment phase market and category ID is defined as the PEA number of the lowest numbered PEA in the assignment phase market followed by the category (e.g., PEA041-M/N or PEA125-P).

If bidder i qualifies for the small business bidding credit and it reached the small markets bidding credit cap, then

- If $k \in SM$,

$$D_{i,k,c} = \frac{GP_{i,k,c}}{\sum_{k' \in SM} \sum_{c' \in \{MN,P\}} GP_{i,k',c'}} \cdot (\$10 \text{ million})$$

- If $k \notin SM$,

$$D_{i,k,c} = \frac{GP_{i,k,c}}{\sum_{k' \in AM \setminus SM} \sum_{c' \in \{MN,P\}} GP_{i,k',c'}} \cdot (FD_i - \$10 \text{ million})$$

That is, the $10 million discount that applies to small markets is apportioned to assignment phase markets that consist of PEAs subject to the small market bidding credit cap proportionally to the bidder's gross payments. The remaining discount (*i.e.*, $FD_i - \$10$ million) is apportioned among the assignment phase markets that consist of PEAs not subject to the small market bidding credit cap.

For each assignment phase market, the calculation is rounded down to the nearest dollar. The slack due to rounding down is then distributed (one dollar at a time) to assignment phase markets and categories based on ascending order of gross payments. Ties are broken based on ascending lexicographic order of assignment phase market and category ID.

In the case of a small business that reached the small markets bidding credit cap, the apportioning of discounts and the distribution of any slack is done separately for the small markets and for the non-small markets.

8.2 Calculation of Gross Per-License Prices

Gross per-license prices are calculated by apportioning assignment phase payments to licenses in proportion to final clock phase prices.

Suppose that the bidder has been assigned a license in category c in PEA j. Let k be the assignment phase market in which PEA j was assigned, that is, $j \in APM(k)$.

The gross pre-license price of a license in category c in PEA j is determined by the following formula:

$$p_{T,i,\{c,j\}} + \frac{p_{T,i,\{c,j\}}}{\sum_{j' \in APM(k)} d_{T,i,\{c,j'\}} \cdot p_{T,i,\{c,j'\}}} \cdot AP_{i,k,c}$$

That is, for each assignment phase market and category, the assignment phase payment is apportioned to the licenses in that assignment phase market and category in proportion to the final clock phase price of each license. Note that if the bidder's assignment payment in an assignment phase market and category is zero, then the gross per-license price of each license it is assigned in that market and category is simply the final clock phase price for that license.

Each license calculation is rounded down to the nearest dollar and then the slack due to rounding down is distributed to licenses (one dollar at a time) based on ascending order of final clock phase prices. Ties are broken based on ascending lexicographic order of license ID. License ID is defined as the PEA number followed by the letter and number representing the block (e.g., PEA261-M1 or PEA001-P2).

8.3 Calculation of Net Per-License Prices

Net per-license prices are calculated by apportioning assignment phase payments and discounts to licenses in proportion to final clock phase prices.

Suppose that the bidder has been assigned a license in PEA j and category c. Let k be the assignment phase market in which PEA j was assigned, that is, $j \in APM(k)$.

The net per-license price of a license in category c in PEA j is determined by the following formula:

$$p_{T,i,\{c,j\}} + \frac{p_{T,i,\{c,j\}}}{\sum_{j' \in APM(k)} d_{T,i,\{c,j'\}} \cdot p_{T,i,\{c,j'\}}} \cdot (AP_{i,k,c} - D_{i,k,c})$$

That is, for each assignment phase market and category, its assignment payment and its discount (see Section 8.1 for how the discount is determined) are apportioned to the licenses in that assignment phase market in proportion to the final clock phase price of each license. Note that if the bidder does not qualify for a bidding credit and its assignment payment in an assignment phase market and category is zero, then the net per-license price of each license it is assigned in that market and category is simply the final clock phase price for that license.

Each license calculation is rounded down to the nearest dollar and then the slack due to rounding down is distributed to licenses (one dollar at a time) based on ascending order of final clock phase prices. Ties are broken based on ascending lexicographic order of license ID.

8.4 Examples

Example 1: Suppose that bidder i qualifies for the rural service provider bidding credit with a bidding credit percentage of 15%, and that:

- Bidder i won one Category M/N block in PEA X and one Category M/N block in PEA Y, that is, $d_{T,i,\{MN,X\}} = d_{T,i,\{MN,Y\}} = 1$, and did not have any other clock phase winnings.
- The final clock phase prices are $p_{T,\{MN,X\}} = 5{,}000{,}000$ and $p_{T,\{MN,Y\}} = 10{,}000{,}000$.
- Bidder i relinquished the following quantities in these PEAs: $\frac{v_{i,X}}{w_X} = 2$ and $\frac{v_{i,Y}}{w_Y} = 0.2$.
- PEAs X and Y were grouped into assignment phase market k for purposes of bidding in the assignment phase.
- The assignment payment of bidder i for Category M/N in assignment phase market k is $AP_{i,k,MN} = 200{,}000$.

Then, the bidder's final gross payment in the assignment phase market is:

$$FGP_i = 5{,}000{,}000 + 10{,}000{,}000 + 200{,}000 = 15{,}200{,}000$$

and its incentive payment is:

$$IP_i = 2 \cdot 5{,}000{,}000 + 0.2 \cdot 10{,}000{,}000 = 12{,}000{,}000$$

The final discount of bidder i is:

$$FD_i = \min\{10{,}000{,}000, (15\%) \cdot (15{,}200{,}000 - 12{,}000{,}000)^+\} = 480{,}000$$

In this example, the bidder has winnings in a single assignment phase market and category. Thus, the bidder's final discount is equal to its discount for that assignment phase market and category, i.e., 480,000.

To calculate the gross per-license price for the license that the bidder is assigned in PEA X, the system performs the following calculation:

$$5{,}000{,}000 + \frac{5{,}000{,}000}{15{,}000{,}000} \cdot 200{,}000 = 5{,}066{,}666.667$$

To calculate the gross per-license price for the license that the bidder is assigned in PEA Y, the system performs the following calculation:

$$10{,}000{,}000 + \frac{10{,}000{,}000}{15{,}000{,}000} \cdot 200{,}000 = 10{,}133{,}333.333$$

Both of these numbers are rounded down to the nearest integer, yielding a slack of $1. The slack is assigned based on ascending order of final clock prices, and thus $1 is assigned to the gross per-license price of PEA X. Thus, the gross per-license prices are 5,066,667 for the license that the bidder is assigned in PEA X, and 10,133,333 for the license that the bidder is assigned in PEA Y.

To calculate the net per-license price for the license that the bidder is assigned in PEA X, the system performs the following calculation:

$$5{,}000{,}000 + \frac{5{,}000{,}000}{15{,}000{,}000} \cdot (200{,}000 - 480{,}000) = 4{,}906{,}666.7$$

To calculate the net per-license price for the license that the bidder is assigned in PEA Y, the system performs the following calculation:

$$10{,}000{,}000 + \frac{10{,}000{,}000}{15{,}000{,}000} \cdot (200{,}000 - 480{,}000) = 9{,}813{,}333.3$$

That is, the assignment payment and the discount are apportioned proportionally to the final clock phase prices of the assigned licenses. Both of these numbers are rounded down to the nearest integer, yielding a slack of $1. The slack is assigned based on ascending order of final clock prices, and thus $1 is assigned to the net per-license price of PEA X. Thus, the net per-license prices are 4,906,667 for the license that the bidder is assigned in PEA X, and 9,813,333 for the license that the bidder is assigned in PEA Y.

<u>Example 2</u>: Bidder i is a 39 GHz incumbent. The bidder qualifies for the small business bidding credit with a bidding credit percentage of $BC_i = 25\%$. The bidder relinquished 1 block equivalent in PEA X and, after round t, the final clock phase price for Category M/N in PEA X is $p_{T,\{MN,X\}} = \$20$ million. Thus, the bidder's incentive payment is $IP_i = \$20$ million. Suppose that bidder i won 1 block for Category M/N in PEA Y, the final clock phase price for Category M/N in PEA Y is $p_{T,\{MN,Y\}} =$ $\$20$ million, and the bidder's assignment payment is $4 million. Thus, the bidder's final gross payment is $FGP_i = \$24$ million. Suppose that PEA Y is a small market and PEA X is not. Then, the bidder's final discount is the minimum of:

i. $FD_i^1 = (25\%) \cdot (\$24\text{ m} - \$20\text{ m})^+ = \$1\text{ m}$; and

ii. $FD_i^2 = min\{\$25\,m, (25\%) \cdot (0 - \$20\text{ m})^+ + min\{\$10\text{ m}, (25\%) \cdot (\$24\text{ m})^+\}\} = \$6\text{ m}$

Thus, the bidder's final discount is $FD_i = \$1$ m.

The gross per-license price for the license that the bidder is assigned in PEA Y is:

$$20{,}000{,}000 + 4{,}000{,}000 = 24{,}000{,}000$$

The net per-license price for the license that the bidder is assigned in PEA Y is:

$$20{,}000{,}000 + (4{,}000{,}000 - 1{,}000{,}000) = 23{,}000{,}000$$