

**Auction 108 Technical Guide
For a Single-Round Auction Format**

1 Introduction

This technical guide sets forth the details of the proposed bidding procedures for Auction 108 under a single-round format as described in the *Auction 108 Comment Public Notice*.¹ Bidding procedures for the simultaneous multiple-round (SMR) auction format on which the Commission also seeks comment in the *Auction 108 Comment Public Notice* are not addressed in this guide.²

Auction 108 will offer approximately 8,300 geographic overlay licenses in the 2.5 GHz band (2496-2690 MHz). For a single-round auction format user-defined package bidding is proposed. Under the single-round format, each bidder could submit bids for individual licenses and package bids for multiple licenses. Furthermore, a bidder could indicate that two or more of its bids are to be treated as mutually exclusive (either/or) by the bidding system when assigning winning bids.

After the single bidding round, the bidding system would use optimization software to determine the value-maximizing combination of (package and individual) bids, taking into account each bidder's mutually exclusive bids. Each winning bidder would pay the sum of its bid amounts for the bids it is assigned, less any applicable bidding credit discount.³

Section 2 describes the bidding requirements. Section 3 describes the calculations for the bidding information shown to bidders. Section 4 describes the calculations for winner determination. Section 5 describes how payments and per-license prices would be calculated at the conclusion of the auction.

2 Bidding Requirements and Definitions

Bidders will submit their bids during a single bidding round.

A bid consists of a set of one or more licenses, and one associated price. The price associated with a bid must be at least the sum of the minimum bid amounts of the licenses in the bid. A bidder can submit at most one bid for a given set of licenses.

A package bid is a bid that includes multiple licenses.

The same license can be included by a bidder in more than one package bid and can also be bid individually.

For the purposes of this auction, a county is considered to be “metropolitan” if it is not subject to the small-market bidding credit cap.⁴

¹ *Auction of Flexible-Use Service Licenses in the 2.5 GHz Band for Next-Generation Wireless Services; Comment Sought on Competitive Bidding Procedures for Auction 108*, AU Docket No. 20-429, Public Notice, FCC 21-14 (Jan. 13, 2021) (*Auction 108 Comment Public Notice*).

² Mathematical details for the SMR auction format are in Attachment B of the *Auction 108 Comment Public Notice*.

³ A bidder will not finally be deemed qualified for a bidding credit until after its qualifications have been approved in the long-form application process. Here, we refer to a bidder that “qualifies for a bidding credit” as a bidder that claimed eligibility for a bidding credit at the short-form stage and whose short-form application qualified the bidder to participate in the auction.

⁴ A county is subject to the small-market bidding credit cap if it is located within a Partial Economic Area (PEA) with a population of 500,000 or less. Thus, a county is considered to be “metropolitan” if it is located within a PEA with a population greater than 500,000.

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A bidder can submit single-county package bids — i.e., packages of multiple licenses in the same county.

In metropolitan counties that are in the same Major Economic Area (MEA) only, a bidder can also submit bids for packages that include any licenses in those counties. The total number of package and/or individual bids that a bidder may submit involving metropolitan counties in an MEA is limited to 250.

A bidder has the option of indicating that two or more of its bids are to be treated as mutually exclusive in the winner determination. All bids in a set of mutually exclusive bids must *either* all involve the same non-metropolitan county *or* all involve only metropolitan counties in the same MEA. Each bid can be included in at most one group of mutually exclusive bids.

A bidder will *not* be allowed to submit a bid or a collection of bids if the maximum number of bidding units that the bidder could win based on those bids would exceed the bidder's eligibility for the auction. Section 3.1 provides the formulation of the optimization problem that is solved to determine the maximum number of bidding units that a bidder could win based on a set of bids.

Example 1: Consider a bidder that does not qualify for a bidding credit discount. Suppose that the bidder has submitted the following bids:

- \$10,000 for License 1
- \$11,000 for License 2

The bidder has not indicated any mutually exclusive bids.

Then, there are four possibilities:

- The bidder wins License 1 for \$10,000.
- The bidder wins License 2 for \$11,000.
- The bidder wins License 1 and License 2 for \$21,000.
- The bidder does not win either license.

Assuming that License 1 and License 2 have 100 bidding units each, the maximum number of bidding units that the bidder could win is 200.

Example 2: Consider a bidder that does not qualify for a bidding credit discount. Suppose that the bidder has submitted the following bids:

- \$10,000 for License 1
- \$11,000 for License 2

The bidder has indicated that these two bids are mutually exclusive.

Then, there are three possibilities:

- The bidder wins License 1 for \$10,000.
- The bidder wins License 2 for \$11,000.
- The bidder does not win either license.

Note that in this example it is not possible for the bidder to win both License 1 and License 2, because the bidder has indicated that its bids are mutually exclusive. Assuming that License 1 and License 2 have 100 bidding units each, the maximum number of bidding units that the bidder could win is 100.

Example 3: Consider a bidder that does not qualify for a bidding credit discount. Suppose that the bidder has submitted the following bids:

- \$20,000 for the package of License 1 and License 2
- \$25,000 for the package of License 3 and License 4
- \$10,000 for License 5

The bidder has also indicated that the first two bids are mutually exclusive.

Then, there are six possibilities:

- The bidder wins License 1 and License 2 for \$20,000.
- The bidder wins License 3 and License 4 for \$25,000.
- The bidder wins License 5 for \$10,000.
- The bidder wins License 1, License 2, and License 5 for \$30,000.
- The bidder wins License 3, License 4, and License 5 for \$35,000.
- The bidder does not win any licenses.

Assuming that License 1 and License 2 have 100 bidding units each, License 3 and License 4 have 120 bidding units each, and License 5 has 50 bidding units, the maximum number of bidding units that the bidder could win is attained when the bidder wins License 3, License 4, and License 5. The maximum number of bidding units is $120+120+50=290$. The bidder will not be allowed to submit these bids if its eligibility for the auction is less than 290.

3 Calculations for Bidding Information

When a bidder uploads a set of bids, the bidding system will use optimization software to calculate the maximum number of bidding units that the bidder could win based on those bids. If the maximum number of bidding units does not exceed the bidder's eligibility and if the bids are otherwise valid bids, the bidding system will accept the bid submission. If the bid submission is accepted, the bidding system will also calculate the maximum dollar amount that the bidder may be obligated to pay based on those bids and provide that information to the bidder.

For instance, if the bidder has eligibility of 100 bidding units, the bidding system would not accept the bid upload submitting the bids of Example 1 but would accept the bid submission of Example 2. If the bidder has eligibility of 200 bidding units or more, the bidding system would accept the bid submission of Example 1 and inform the bidder that (i) the bidder could win a maximum of 200 bidding units, and (ii) the maximum dollar amount that the bidder may be obligated to pay is \$21,000. For the bid submission of Example 2, the bidding system would inform the bidder that (i) the bidder could win a maximum of 100 bidding units, and (ii) the maximum dollar amount that the bidder may be obligated to pay is \$11,000.

This section provides the formulations of these two optimization problems for a given bidder i , using the following notation:

- K denotes the set of all licenses.
- u_k denotes the number of bidding units for license k .
- S denotes a set of licenses. For each license k , S_k denotes the indicator variable of whether license k is in set S . That is, $S_k = 1$ if $k \in S$, and $S_k = 0$ if $k \notin S$.
- F_i denotes the set of bids of bidder i . Each $S \in F_i$ is a set of licenses for which bidder i submitted a bid. Note that S may consist of a single license.
- $b_i(S)$ denotes the bid amount of bidder i for the set of licenses $S \in F_i$.
- M_i denotes the number of groups of mutually exclusive bids submitted by bidder i . If bidder i did not indicate any of its bids as mutually exclusive, then $M_i = 0$.
- $G_{i,j}$ denotes the j -th group of mutually exclusive bids submitted by bidder i . This is defined for $j \in \{1, 2, \dots, M_i\}$.

3.1 Maximum Number of Bidding Units for Bidder i

To determine the maximum number of bidding units that bidder i could win based on the set of bids it submitted, the bidding system will use optimization software to calculate the maximum number of bidding units that can be won by the bidder subject to the constraints that (1) each license is assigned at most once, and (2) for each group of bids that the bidder indicated as mutually exclusive, the bidder is assigned at most one of those bids. This optimization problem considers only the bids of bidder i .

The optimization problem can be formulated as follows:

$$\max \sum_{k \in K} u_k \sum_{S \in F_i} S_k \cdot X_i(S)$$

Subject to:

$$\sum_{S \in F_i} S_k \cdot X_i(S) \leq 1 \quad \forall k \in K \quad (1)$$

$$\sum_{S \in G_{i,j}} X_i(S) \leq 1 \quad \forall j \in \{1, 2, \dots, M_i\} \quad (2)$$

$$X_i(S) \in \{0, 1\} \quad \forall S \in F_i \quad (3)$$

Variable Definition:

$X_i(S)$ is a binary decision variable which has a value of 1 if bidder i wins the bid with the set of licenses S , and 0 otherwise. This variable is defined for all $S \in F_i$.

Objective:

The objective function is equal to the sum of bidding units across all licenses that are won by bidder i .

Explanation of Constraints:

- Constraint (1) ensures that each license is assigned at most once.

- Constraint (2) ensures that for each group of bids that bidder i indicated as mutually exclusive, the bidder is assigned at most one of those bids.
- Constraint (3) states that each decision variable $X_i(S)$ can be either equal to 0 or 1.

3.2 Maximum Dollar Amount of the Bids of Bidder i

To determine the maximum dollar amount that bidder i may be obligated to pay, the bidding system will use optimization software to determine the value-maximizing combination of (package and individual) bids of bidder i subject to the constraints that (1) each license is assigned at most once, and (2) for each group of bids that the bidder indicated as mutually exclusive, the bidder is assigned at most one of those bids. This optimization problem considers only the bids of bidder i .

The optimization problem can be formulated as follows:

$$\max \sum_{S \in F_i} b_i(S) \cdot X_i(S)$$

Subject to:

$$\sum_{S \in F_i} S_k \cdot X_i(S) \leq 1 \quad \forall k \in K \quad (1)$$

$$\sum_{S \in G_{i,j}} X_i(S) \leq 1 \quad \forall j \in \{0,1, \dots, M_i\} \quad (2)$$

$$X_i(S) \in \{0,1\} \quad \forall S \in F_i \quad (3)$$

The objective function is equal to the sum of bid amounts across all bids assigned to bidder i . The variable definition and the constraints are identical to the problem of Section 3.1.

4 Winner Determination

After the single bidding round, the system will determine the winning bids. In particular, the bidding system will use optimization software to determine the value-maximizing combination of package and individual bids, taking into account each bidder's mutually exclusive either/or bids.⁵ Ties, if any, are broken by including pseudorandom numbers in an optimization.

To mathematically formulate the winner determination problem,⁶ the following notation is used in addition to the notation introduced in Section 3:

- N denotes the set of bidders.

⁵ Because there is a very small but positive probability that the optimization software will be unable to provide an exact solution to the problem of determining the value-maximizing combination of bids within a reasonable amount of time, the *Auction 108 Comment Public Notice* has proposed an “escape clause.” Under the proposed escape clause, if the optimization software does not yield an exact solution within 48 hours, then the winning set of bids would be determined by the best solution identified to that point. In any case, winning bidders would pay the amounts of their winning bids, consistent with the pay-as-bid pricing rule.

⁶ The formulation in this section considers all licenses in one optimization problem. However, it is equivalent to solve a separate optimization problem for each MEA (considering all bids for metropolitan counties) and for each non-metropolitan county.

- b denotes the set of bid amounts.

Variable Definition:

$X_i(S)$ is a binary decision variable which is set equal to 1 if bidder i wins package S , and 0 otherwise. This variable is defined for all $i \in N$ and all $S \in F_i$.

4.1 Maximum Sum of Bid Amounts

$$r(b) = \max \sum_{i \in N} \sum_{S \in F_i} b_i(S) \cdot X_i(S)$$

Subject to:

$$\sum_{i \in N} \sum_{S \in F_i} S_k \cdot X_i(S) \leq 1 \quad \forall k \in K \quad (1')$$

$$\sum_{S \in G_{i,j}} X_i(S) \leq 1 \quad \forall i \in N, \forall j \in \{0, 1, \dots, M_i\} \quad (2')$$

$$X_i(S) \in \{0, 1\} \quad \forall i \in N, \forall S \in F_i \quad (3')$$

Explanation of Objective:

The objective function is equal to the sum of bid amounts of an assignment, across all bidders.

Explanation of Constraints:

- Constraint (1') ensures that each license is assigned at most once.
- Constraint (2') ensures that, for each bidder, for each group of bids that the bidder indicated as mutually exclusive, the bidder is assigned at most one of those bids.
- Constraint (3') states that each decision variable $X_i(S)$ can be either equal to 0 or 1.

Note that constraints (1') – (3') above are the same as constraints (1) – (3) of Section 3, except that they consider the bids of all bidders.

4.2 Tie-breaking

For every set S and every bidder $i \in N$, the bidding system generates a pseudorandom number $\xi_i(S)$ drawn uniformly at random from the set $\{1, 2, \dots, 10^7\}$. The bidding system then solves an optimization problem to find the assignment that maximizes the sum of pseudorandom numbers among all assignments that satisfy constraints (1') – (3') of Section 4.1 such that the sum of bid amounts is equal to $r(b)$. In particular, the optimization problem is formulated as follows:

$$\max \sum_{i \in N} \sum_{S \in F_i} \xi_i(S) \cdot X_i(S)$$

Subject to:

$$\sum_{i \in N} \sum_{S \in F_i} S_k \cdot X_i(S) \leq 1 \quad \forall k \in K \quad (1')$$

$$\sum_{S \in G_{i,j}} X_i(S) \leq 1 \quad \forall i \in N, \forall j \in \{0, 1, \dots, M_i\} \quad (2')$$

$$X_i(S) \in \{0, 1\} \quad \forall i \in N, \forall S \in F_i \quad (3')$$

$$\sum_{i \in N} \sum_{S \in F_i} b_i(S) \cdot X_i(S) \geq r(b) \quad (4')$$

Constraints (1') through (3') are the same as in the optimization of Section 4.1.

Explanation of New Constraint:

Constraint (4') states that the sum of bid amounts must be greater than or equal to the result of the optimization of Section 4.1.

5 Payments and Per-License Prices

Each winning bidder will be obligated to pay the sum of its bid amounts for all the bids it is assigned, less any applicable bidding credit discount.

Because the bid amounts for package bids and the discount for a winning bidder with a bidding credit apply on an aggregate basis, rather than for individual licenses, the bidding system will also calculate a gross and net per-license price for each license won. Such individual prices may be needed in the event that a licensee subsequently incurs license-specific obligations, such as unjust enrichment payments.

Section 5.1 describes how the bidding credit discount and the net payment is calculated for a bidder that qualifies for a bidding credit. Section 5.2 describes how gross and net per-license prices are calculated.

5.1 Bidding Credit Discount and Net Payment

This section uses the following notation:

- BC_i denotes the bidding credit percentage of bidder i .
- P_i denotes the sum of bid amounts for all bids that bidder i is assigned.
- P_i^{SM} denotes the sum of bid amounts for all bids involving counties subject to the small-market bidding credit cap (i.e., non-metropolitan counties) that bidder i is assigned.
- P_i^{NSM} denotes the sum of bid amounts for all bids involving counties not subject to the small-market bidding credit cap (i.e., metropolitan counties) that bidder i is assigned. By definition, $P_i = P_i^{NSM} + P_i^{SM}$.

Rural Service Provider Bidding Credit. If bidder i qualifies for the rural service provider bidding credit,

Its *uncapped discount* is:

$$BC_i \cdot P_i$$

Its *discount* is:

$$\min\{\$10 \text{ million}, BC_i \cdot P_i\}$$

This is equal to the sum of the bidder's winning bid amounts multiplied by its bidding credit percentage and then capped at \$10 million.

Small Business Bidding Credit. If bidder i qualifies for the small business bidding credit,

Its *uncapped discount in small markets* is:

$$BC_i \cdot P_i^{SM}$$

Note that P_i^{SM} is across all of the bidder's winning bids for counties subject to the small-market bidding credit cap. The uncapped discount in small markets is calculated by multiplying P_i^{SM} by its bidding credit percentage.

Its *uncapped discount* (across all markets) is:

$$BC_i \cdot P_i$$

Its *discount* (across all markets) is:

$$\min\{\$25 \text{ million}, BC_i \cdot P_i^{NSM} + \min\{\$10 \text{ million}, BC_i \cdot P_i^{SM}\}\}$$

This calculation first caps the bidder's discount in small markets at \$10 million, then adds the bidder's discount from all other counties (*i.e.*, counties that are not subject to the small-market bidding credit cap) and caps the sum at \$25 million.

All bidding credit discounts are rounded to the nearest dollar. Rounding is only done at the very end of a given calculation, that is, after performing any summations and/or minimizations in a formula.

Net Commitment. A bidder's *net payment* is equal to its the sum of its winning bid amounts minus its discount.

5.2 Per-License Prices

Once winning bids have been determined, the bidding system will determine a net and gross price for each license that was won by a bidder by apportioning package bid amounts and bidding credit discounts (only applicable for the net price) across all the licenses that the bidder won.

The gross price for a license that was won as part of a bid that only contained that license will be equal to the bidder's bid amount for that license. To calculate the gross price for a given license that was won as part of a package bid, the bidding system will apportion the bidder's bid amount for the package to the licenses in the package in proportion to the minimum bid amounts of those licenses.

For a license won by a bidder that does not qualify for a bidding credit, the net price is simply equal to the gross price of the license.

To calculate the net price for a bidder that qualifies for a bidding credit, the bidding system will apportion the bidder's overall discount to licenses in proportion to the minimum bid amounts of the licenses that the bidder is assigned.