Auction 110 Assignment Phase Technical Guide

1 Introduction

This technical guide sets forth the details of the bidding procedures for the second phase—the assignment phase—of Auction 110 as described in the *Auction 110 Procedures Public Notice*.[1] The assignment phase offers winners of generic spectrum blocks in the clock phase of Auction 110 the opportunity to bid for specific frequency assignments. The bidding system will determine specific license assignments and any payment, above the final clock phase price, that a winning bidder will pay for the assignment.

Pursuant to the *3.45 GHz Second Report and Order*, the 3.45–3.55 GHz band will be reconfigured and licensed in uniform 10-megahertz blocks in each of the 406 PEAs in the contiguous United States.[2] In most PEAs, new licensees generally will have unrestricted use of all ten frequency blocks. In other areas, licensees must coordinate with incumbent federal operations in the band. In some of the PEAs where coordination is required, all ten blocks will be subject to the same requirements. In others, the requirements may vary depending upon the frequency block—specifically, in 72 PEAs, four blocks (A through D) will be subject to different requirements than the remaining six blocks (E through J). Finally, in three PEAs (PEAs 041, 044, and 227), eight blocks (A through H) will be subject to restrictions resulting from the Special Temporary Authority (STA) granted to Lockheed Martin Corporation, whereas the remaining two blocks (I and J) will not be subject to any restrictions.[3]

The Commission has established categories for bidding as follows: in the PEAs where all ten blocks are the same—*i.e.*, all ten generally are unrestricted or all ten are subject to the same requirements—the ten generic blocks will be considered Category 1, or "Cat1," blocks. In the PEAs subject to cooperative sharing requirements or restrictions where the requirements differ according to the frequency, blocks A through D will be considered Category 1, or "Cat1," while blocks E through J will be considered Category 2, or "Cat2." In the three PEAs subject to restrictions resulting from the STA granted to Lockheed Martin Corporation, blocks A through H will be considered Category 1, or "Cat1," while blocks I and J will be considered Category 2, or "Cat2."

The assignment phase is designed to promote two major goals. One of these goals is to make bidding relatively easy even though the underlying allocation problem is complex. The procedure promotes simplicity in several ways. First, to reduce the total number of bids that each bidder must make, it groups together non-top 20 PEAs within a region under certain conditions. Second, to simplify bidding strategy for bidders, it uses a *second-price* type of pricing rule that encourages bidders to bid according to their actual values for different frequencies while ensuring that there is no bidder or group of bidders willing to pay more for an alternative feasible assignment. Third, bidding in the assignment phase is voluntary. A winner of generic blocks in the clock phase does not need to make any additional bids—or make any additional payments—in the assignment phase. This makes bidding easier in both the assignment phase and the clock phase of the auction, because bidders in the clock phase will know that they need not pay more than the prices bid in the clock phase.

[1] *See Auction of Flexible-Use Licenses in the 3.45-3.55 GHz Band for Next Generation Wireless Services; Notice and Filing Requirements, Minimum Opening Bids, Upfront Payments, and Other Procedures for Auction 110, Bidding in Auction 110 Scheduled to Begin October 5, 2021*, AU Docket No. 21-62, Public Notice, DA 21-655 (OEA/WTB June 9, 2021).

[2] *See generally Facilitating Shared Use in the 3100-3550 MHz Band*, WT Docket No. 19-348, Second Report and Order, Order on Reconsideration, and Order of Proposed Modification, FCC 21-32 (Mar. 18, 2021) (*3.45 GHz Second Report & Order*).

[3] *Lockheed Martin Request for Part 90 Special Temporary Authority to Operate Two Radiolocation Service Sites in the 3.45 GHz Band*, ULS File No. 0009581172, Order, DA 21-693 (WTB June 16, 2021).

A second, equally important goal is to promote efficient and intensive use of the spectrum. To achieve that, the assignment phase rules ensure that each clock phase winner is assigned contiguous frequencies within each category in each PEA, even if the clock phase winner does not bid in the assignment phase. Moreover, in PEAs with both Cat1 and Cat2 blocks, if one or more bidders have won blocks in both categories in the clock phase, the assignment phase rules ensure that one of those bidders will be assigned licenses that are contiguous across the categories.

2 Assignment Rounds

The assignment phase consists of a series of *assignment rounds.* In each assignment round, frequencies are assigned in up to six *assignment phase markets*, with each assignment phase market consisting of either a single PEA or a group of PEAs. Winning bidders from the clock phase that have a preference for specific license frequencies submit sealed bids for those frequencies, separately for each category. Once an assignment round concludes, the system will consider the bids and assign specific frequencies to each winning bidder from the clock phase. Additional optimizations are solved to determine each bidder's assignment payment, which will be equal to or less than the bidder's bid value for the assignment.

The bidding system will determine whether to group PEAs into a single assignment phase market according to the rule detailed in Section 2.1 below.

2.1 Grouping PEAs into a Single Assignment Phase Market

A set of PEAs *P* will be grouped into one *assignment phase market* if all of the following four conditions are met:

(1) The PEAs in *P* are all in the same Regional Economic Area Grouping ("REAG")[4] and not in the top-20 PEAs;[5]

(2) Either all PEAs in *P* are not subject to the small market bidding credit cap[6] or all PEAs in *P* are subject to the small market bidding credit cap;

(3) The PEAs in *P* have the same categories and the same supply for each category; and

(4) For each PEA in *P*, the same bidders won the same number of blocks in each category.

Because of this grouping of PEAs, the number of assignment phase markets will be smaller than or equal to the number of PEAs in the auction.

Example 1: REAG Grouping

PEA X, PEA Y, and PEA Z are all in REAG 1. These PEAs are not top-20 PEAs and are not subject to the small market bidding credit cap. Each of these PEAs has 10 Cat1 blocks.

In each of these PEAs:

- Bidder 1 won 4 Cat1 blocks in the clock phase.

[4] The six REAGs are: Northeast, Southeast, Great Lakes, Mississippi Valley, Central, and West.

[5] The top-20 PEAs are PEAs 1–20.

[6] PEAs that are subject to the small market bidding credit cap in Auction 110 are those PEAs with a population of 500,000 or less, which correspond to PEAs 118–211, 213–263, 265–297, 299–359, and 361–411. *See Updating Part 1 Competitive Bidding Rules et al.*, WT Docket Nos. 14-170, 05-211, GN Docket No. 12-268, RM-11395, Report and Order, Order on Reconsideration of the First Report and Order, Third Order on Reconsideration of the Second Report and Order, Third Report and Order, 30 FCC Rcd 7493, 7546-47, paras. 127-28 (2015).

- Bidder 2 won 4 Cat1 blocks in the clock phase.
- Bidder 3 won 2 Cat1 blocks in the clock phase.

Then PEA X, PEA Y, and PEA Z will be grouped and treated as a single combined market for the assignment phase.

<u>Example 2</u>: Not Possible to Group

PEA X and PEA Y are both in REAG 2 and are not in the top-20 PEAs. Neither of these PEAs is subject to the small market bidding credit cap. Each of these PEAs has 4 Cat1 blocks and 6 Cat2 blocks.

The winners in these two PEAs are as follows:

In PEA X:

- Bidder 1 won 2 Cat1 blocks in the clock phase.
- Bidder 2 won 2 Cat1 blocks and 2 Cat2 blocks in the clock phase.
- Bidder 3 won 4 Cat2 blocks in the clock phase.

In PEA Y:

- Bidder 1 won 2 Cat1 blocks in the clock phase.
- Bidder 2 won 2 Cat1 blocks and 2 Cat2 blocks in the clock phase.
- Bidder 4 won 4 Cat2 blocks in the clock phase.

Then, PEA X and PEA Y will not be grouped, because they do not have identical clock phase winners. That is, Bidder 3 won in PEA X, whereas Bidder 4 won in PEA Y.

2.2 Sequencing of Assignment Rounds

The assignment phase begins with assignment rounds for the top-20 PEAs (PEAs 1-20). The top-20 PEAs are ordered in descending order of population (pops), and bidding is conducted for a single assignment phase market per round, sequentially. Note that there is no grouping for the top-20 PEAs.

After bidding has been conducted for the top-20 PEAs, bidding is conducted simultaneously for PEAs in the six REAGs, but in descending order of PEA (or PEA group) pops within each REAG. That is, bidding may be conducted for up to six assignment phase markets at the same time, in order to speed up the assignment phase. The rounds continue until all assignment phase markets are assigned.

If an assignment phase market consists of multiple PEAs, its pops will be set equal to the sum of the pops of the PEAs that it comprises for purposes of determining the sequencing.

Before bidding for the assignment phase starts, the bidding system will inform bidders about which PEAs have been grouped and the sequencing of assignment rounds.

The following tables show two examples of the sequencing of assignment phase markets. In the first example, there is no grouping, that is, each assignment phase market consists of a single PEA. In the second example, some assignment phase markets consist of multiple PEAs and, as a result, there are fewer assignment rounds.

Table 1: Sequencing of assignment phase markets with no grouping

Round	PEA					
1	001					
2	002					
…	…					
20	020					
	REAG 1	**REAG 2**	**REAG 3**	**REAG 4**	**REAG 5**	**REAG 6**
21	041	021	023	024	028	022
22	044	029	025	030	035	026
…	…	…	…	…	…	…

Table 2: Sequencing of assignment phase markets with grouping

Round	PEA(s)					
1	001					
2	002					
…	…					
20	020					
	REAG 1	**REAG 2**	**REAG 3**	**REAG 4**	**REAG 5**	**REAG 6**
21	041; 044	021	023	024	028	022; 034
22	048	029; 040	025	030	035	026
…	…	…	…	…	…	…

As illustrated in the tables above, after bidding for the top-20 PEAs is finished, bidding for multiple assignment phase markets will be conducted in the same round.

3 Bidding

3.1 Bidding Options

For each category and assignment phase market, the bidding system will determine all contiguous assignment options where the number of licenses per PEA is equal to the number of blocks that the winner has won in the clock phase in that category and assignment phase market. This set is referred to as the *bidding options* of the bidder. Note that the bidding options of a bidder do not depend on the clock phase winnings of other bidders.

Example 3: Consider an assignment phase market that consists of PEAs with a single category (Cat1). A bidder won 3 Cat1 blocks in each of these PEAs. Then, the bidder will have the following 8 bidding options: ABC, BCD, CDE, DEF, EFG, FGH, GHI, and HIJ.

Example 4: Consider an assignment phase market that consists of PEAs with 4 Cat1 blocks (A through D) and 6 Cat2 blocks (E through J). Then:

- A bidder that won 1 Cat1 block will have 4 bidding options: A, B, C, and D.
- A bidder that won 2 Cat2 blocks will have 5 bidding options: EF, FG, GH, HI, and IJ.

- A bidder that won 1 Cat1 block and 3 Cat2 blocks will have two sets of bidding options. The bidding options for Cat1 are: A, B, C, and D. The bidding options for Cat2 are: EFG, FGH, GHI, and HIJ.

The bidder can bid on any or all of its bidding options. Note that in certain instances, however, some of the bidding options cannot be assigned. For instance, in a PEA with 10 Cat1 blocks, if one bidder won 2 Cat1 blocks and two other bidders won 4 Cat1 blocks each, the first bidder will have BC as one of its bidding options, but cannot be assigned BC. The reason is that, if it were assigned BC, it would not be possible to assign contiguous spectrum to the other bidders. The bidding options of a bidder are not limited only to the options that can be won by the bidder, because limiting the bidding options in that way may permit a bidder to infer the clock phase winnings of other bidders.

3.2 Automatic Assignments and Pre-Assignments

If a bidder has won all the available blocks in a category in a given assignment phase market, the bidder will be automatically assigned those blocks and will not be able to submit bids for this category and assignment phase market. Because there is an aggregation limit of four blocks in Auction 110, automatic assignments can only occur in the following two cases: for PEAs with four Cat1 blocks where a single bidder won all four Cat1 blocks, and for PEAs with two Cat2 blocks where a single bidder won both Cat2 blocks.

If a PEA has no winners from the clock phase (all blocks remain FCC-held), all the blocks in that PEA will be pre-assigned to the FCC and there will not be a round held for that PEA. In a PEA with two categories, if all the available blocks in one category were won by a single bidder and all the blocks in the other category remain FCC-held, then all the blocks in that PEA will be pre-assigned and there will not be a round held for that PEA.

3.3 Bidding Rules

A bidder may specify a bid value for each of its bidding options for an assignment phase market. The bidder bids for a bidding option by specifying a bid amount for that option. The bid amount must be non-negative, must be a multiple of $100, and cannot exceed $999,999,900.

A bidder that does not have clock phase winnings in a PEA will not have any bidding options in the corresponding assignment phase market and thus cannot submit bids for that market.

If an assignment phase market is a group of PEAs, then each bidder has the same clock phase winnings in each of those PEAs (because of the grouping rule described in Section 2.1). By specifying a bid value for a bidding option, the bidder indicates the maximum amount that it is willing to pay, in total, to be assigned that option in all those PEAs. A bidder will not be able to bid for different frequency assignments in the various PEAs within a group.

A clock phase winner is not required to bid in the assignment phase. In particular, a bidder may not wish to bid if it is indifferent among all assignments that it may get. The bidding system will consider a bid value of zero for any set of licenses for which a bidder submits no bid.

3.4 Winning Assignments and Payments

After bidding in an assignment round concludes, the bids are processed to determine the winning assignments and the payments for that round. For each assignment phase market of the round, each bidder is then informed about its winning assignment and its assignment payment for each category in that assignment phase market. This information is disclosed to the bidder before the next assignment round starts.

4 Overview of Assignment and Assignment Payment Determination

After each assignment round, the bidding system will determine the assignment and the assignment payments for each assignment phase market and category in that round. This determination will be done separately for each assignment phase market.

The winning assignment is generally determined by maximizing the sum of bid amounts while ensuring that each bidder is assigned contiguous spectrum within each category. The assignment payments are then calculated consistent with a generalized "second price" approach—that is, the winner will pay a price that would be just sufficient to result in the bidder receiving that same winning frequency assignment while ensuring that no group of bidders is willing to pay more for an alternative assignment where each bidder is assigned contiguous spectrum within that category.

For an assignment phase market consisting of PEAs with one category (*i.e.*, Cat1), the bidding system will directly determine the assignment and the assignment payments as described in Sections 5 and 6 respectively.

For an assignment phase market consisting of PEAs with two categories (*i.e.*, Cat1 and Cat2), the bidding system will first determine whether there are one or more bidders with winnings in both categories. If there are no such bidders, the bidding system will determine, separately for each category, the assignment and the assignment payments as described in Sections 5 and 6 respectively.

For an assignment phase market consisting of PEAs with two categories where one or more bidders have won blocks in both categories in the clock phase, the bidding system will ensure that one of those bidders will be assigned licenses that are contiguous across the two categories. In particular:

- If only one bidder has clock phase winnings in both categories, then that bidder will be assigned licenses that are contiguous across the two categories and its assignment payment will be zero.
- If two or more bidders have won blocks in both categories, the bidding system will consider the sum of each such bidder's bid for its Cat1 option that includes the highest-frequency block and its bid for the Cat2 option that includes the lowest-frequency block.[7] The bidder with the highest bid total[8] will be assigned licenses that are contiguous across the two categories. The bidder's assignment payment will be the price of the bidder with the second-highest total bid. The bidding system will then determine the bidder's assignment payment for each category[9] by apportioning the total payment proportionally to the bidder's bid for its Cat1 option that includes the highest-frequency block and its bid for the Cat2 option that includes the lowest-frequency block.[10]

Once the bidding system has determined which bidder is assigned contiguous spectrum across Cat1 and Cat2, it will then determine (1) the assignment and assignment payments for Cat1, following the approach

[7] In the case of a PEA with four Cat1 blocks and six Cat2 blocks, the highest-frequency block in Cat1 is D and the lowest-frequency block in Cat2 is E. In the case of a PEA with eight Cat1 blocks and two Cat2 blocks, the highest-frequency block in Cat1 is H and the lowest-frequency block in Cat2 is I.

[8] In the case of a tie, the bidding system will use pseudorandom numbers to break ties. As described in Section 5.2, a pseudorandom number is generated for every bidding option of a bidder. If two or more of the bidders with winnings in both categories have the highest bid total, the tie will be broken by selecting the bidder with the highest sum of pseudorandom numbers for its Cat1 option that includes the highest-frequency block and its Cat2 option that includes the lowest-frequency block.

[9] An assignment payment for each category is needed for the per-license price calculations described in Section 10.

[10] The results of these calculations will be rounded down and any slack due to rounding will be distributed to the bidder's Cat1 payment.

described in Sections 5 and 6, but excluding the Cat1 licenses that have already been assigned to that bidder, and (2) the assignment and assignment payments for Cat2, following the approach described in Sections 5 and 6, but excluding the Cat2 licenses that have already been assigned to that bidder.

Example 5: Consider an assignment phase market consisting of PEAs with four Cat1 blocks and six Cat2 blocks. Four bidders have won blocks in the clock phase:

- Bidder 1 won 1 Cat1 block and 2 Cat2 blocks
- Bidder 2 won 2 Cat1 blocks and 2 Cat2 blocks
- Bidder 3 won 1 Cat1 block
- Bidder 4 won 2 Cat2 blocks

Table 3 below summarizes the bids for this example.

Table 3. Bids for Example 5

Bidders	*Bidder 1*		*Bidder 2*		*Bidder 3*		*Bidder 4*	
Blocks won	*1 Cat1, 2 Cat2*		*2 Cat1, 2 Cat2*		*1 Cat1*		*2 Cat2*	
Cat1 bids	A:	$0	**AB:**	**$0**	A:	$0		
	B:	$300	BC:	$500	B:	$0		
	C:	$300	CD:	$100	**C:**	**$1,000**		
	D:	**$400**			D:	$1,000		
Cat2 bids	**EF:**	**$600**	EF:	$100			EF:	$0
	FG:	$500	FG:	$100			FG:	$0
	GH:	$0	**GH:**	**$100**			GH:	$0
	HI:	$0	HI:	$0			HI:	$0
	IJ:	$0	IJ:	$0			**IJ:**	**$0**

Since two bidders (Bidder 1 and Bidder 2) have won blocks in both categories, the bidding system will first determine which of those two bidders will be assigned contiguous spectrum across Cat1 and Cat2.

Determining which bidder will be assigned contiguous spectrum: For each bidder with winnings in both categories (Bidder 1 and Bidder 2), the bidding system will consider the sum of its bid for its Cat1 option that includes the highest-frequency block (D) and its bid for the Cat2 option that includes the lowest-frequency block (E). For Bidder 1, the relevant bids are $400 for D and $600 for EF, so the total is $400 + $600 = $1000. For Bidder 2, the relevant bids are $100 for CD and $100 for EF, so the total is $100 + $100 = $200. Thus, Bidder 1 has the highest bid total and will be assigned contiguous licenses across Cat1 and Cat2. Specifically, Bidder 1 will be assigned license D in Cat1 and licenses E and F in Cat2.

Determining the assignment payments of the bidder assigned contiguous spectrum: The assignment payment of Bidder 1 across the two categories equals the total bid amount of Bidder 2 for options CD and EF and thus is $200. The bidding system will apportion this total pro rata to determine the assignment payments of Bidder 1 for each category:

- For Cat1, the assignment payment of Bidder 1 is: 200*400/1000 = $80
- For Cat2, the assignment payment of Bidder 1 is: 200*600/1000 = $120

Determining the assignment and the assignment payments for the remaining Cat1 licenses: Since license D has already been assigned to Bidder 1, the bidding system will determine the assignment of the remaining Cat1 licenses (*i.e.*, A, B, and C) to Bidders 2 and 3, taking into account the bids of Bidder 2 for options AB and BC and the bids of Bidder 3 for options A, B, and C. The sum of bid amounts is maximized when Bidder 2 is assigned AB and Bidder 3 is assigned C. Using the second-price type of pricing rule described in Section 6, the system will determine that, for Cat1, the assignment payment of Bidder 2 is $0 and the assignment payment of Bidder 3 is $500.

Determining the assignment and the assignment payments for the remaining Cat2 licenses: Since licenses E and F have already been assigned to Bidder 1, the bidding system will determine the assignment of the remaining Cat2 licenses (*i.e.*, G, H, I, and J) to Bidders 2 and 4, taking into account the bids of Bidder 2 for options GH, HI, and IJ, and the bids of Bidder 4 for options GH, HI, and IJ. The sum of bid amounts is maximized when Bidder 2 is assigned GH and Bidder 4 is assigned IJ. Since all bid amounts of Bidder 4 are $0, the bidding system will determine that, for Cat2, the assignment payment of Bidder 2 is $0 and the assignment payment of Bidder 4 is $0.

5 Assignment Determination for a Given Category and Assignment Phase Market

For a given category in a given assignment phase market, an assignment is *feasible* if:

(1) Each bidder is assigned one of its bidding options; and

(2) Any FCC-held licenses are contiguous.

For an assignment phase market consisting of PEAs with two categories, the winning assignment is determined separately for each category by maximizing the sum of bid values for the corresponding category (after excluding any blocks that have already been assigned to a bidder with winnings in both categories, per Section 4). Ties, if any, are broken by including pseudorandom numbers in an optimization.

Specifically, the assignment determination is done by solving two optimization problems for each category in each assignment phase market. The first optimization problem finds the maximum sum of bid values among all feasible assignments. The bidding system then solves another optimization problem using randomly generated numbers to break ties, if any. The solution to the latter optimization is selected as the winning assignment.

To mathematically formulate the assignment determination for a given category in a given assignment phase market, the following notation is used:

- N denotes the set of bidders in that category and assignment phase market, that is, the set of clock phase winners with winnings in this particular category and assignment phase market. In the case of an assignment phase market with two categories where one or more bidders won blocks in both categories, the set N does not include the bidder that has already been assigned contiguous spectrum across the two categories, per Section 4.
- The FCC is referred to as bidder 0, and $N \cup \{0\}$ is used to denote the set of bidders and the FCC.
- K denotes the set of licenses for the category in the assignment phase market. For an assignment phase market with a single category (*i.e.*, Cat1), K consists of licenses A, B, C, …, J. For an assignment phase market with two categories: for Cat1, K consists of licenses A, B, C, and D; for Cat2, K consists of licenses E, F, G, H, I, and J. In the case of an assignment phase market with two categories where one or more bidders won blocks in both categories, the set K does not include the blocks that have already been assigned to the bidder that was assigned contiguous spectrum across the two categories, per Section 4.
- S denotes a set of licenses. For each license k, S_k denotes the indicator variable of whether license k is in set S. That is, $S_k = 1$ if $k \in S$, and $S_k = 0$ if $k \notin S$.

- m_i is the number of blocks per PEA won by bidder i in the category and assignment phase market of interest.
- m_0 is the number of FCC-held blocks per PEA in the category and assignment phase market of interest.
- F_i denotes the set of bidding options for bidder i. That is, F_i will consist of all sets $S \subseteq K$ with m_i contiguous licenses (see Section 3 for examples).
- F_0 consists of all sets $S \subseteq K$ with m_0 contiguous licenses. This set gives the possible assignments for any FCC-held blocks.
- $b_i(S)$ denotes the bid value of bidder i for set $S \in F_i$.
- b denotes the set of bid values.

Variable Definition:

$X_i(S)$ is a binary decision variable which has a value of 1 if exactly the licenses of set S are assigned to bidder i, and 0 otherwise. This variable is defined for all $i \in N \cup \{0\}$. Thus, $X_0(S) = 1$ if the set of licenses assigned to the FCC is S.

5.1 Maximum Sum of Bid Values

$$r(b) = max \sum_{i \in N} \sum_{S \in F_i} b_i(S) \cdot X_i(S)$$

Subject to:

$\sum_{i \in N \cup \{0\}} \sum_{S \in F_i} S_k \cdot X_i(S) = 1$	$\forall k \in K$	(1)
$\sum_{S \in F_i} X_i(S) = 1$	$\forall i \in N \cup \{0\}$	(2)
$X_i(S) \in \{0,1\}$	$\forall i \in N \cup \{0\}, \forall S \in F_i$	(3)

The objective function is equal to the sum of bid values of an assignment, across all bidders.

Explanation of Constraints:

- Constraint (1) ensures that each license is assigned exactly once, either to one of the bidders or to the FCC.
- Constraint (2) ensures that each bidder is assigned exactly one of its bidding options and that the set of licenses assigned to the FCC is contiguous.
- Constraint (3) states that each decision variable $X_i(S)$ can be either equal to 0 or 1.

5.2 Tie-Breaking

For every bidder $i \in N$ and every bidding option $S \in F_i$, the bidding system generates a pseudorandom number $\xi_i(S)$ drawn uniformly at random from the set $\{0, 1, 2, \ldots, 2^{24} - 1\}$. The bidding system then solves an optimization problem to find the assignment that maximizes the sum of pseudorandom numbers among all assignments that satisfy constraints (1) through (3) of Section 5.1 such that the sum of bid values is equal to $r(b)$. In particular, the optimization problem is formulated as follows:

$$max \sum_{i \in N} \sum_{S \in F_i} \xi_i(S) \cdot X_i(S)$$

Subject to:

$\sum_{i \in N \cup \{0\}} \sum_{S \in F_i} S_k \cdot X_i(S) = 1$	$\forall k \in K$	(1)
$\sum_{S \in F_i} X_i(S) = 1$	$\forall i \in N \cup \{0\}$	(2)
$X_i(S) \in \{0,1\}$	$\forall i \in N \cup \{0\}, \forall S \in F_i$	(3)
$\sum_{i \in N} \sum_{S \in F_i} b_i(S) \cdot X_i(S) \geq r(b)$		(4)

Constraints (1) through (3) are the same as in the optimization of Section 5.1.

Explanation of New Constraint:

Constraint (4) states that the sum of bid values must be greater than or equal to the result of the optimization of Section 5.1.

6 Assignment Payment Determination for a Given Category and Assignment Phase Market

A bidder's assignment payment for the set of licenses it is assigned for a category in an assignment phase market is an additional payment amount above its clock phase payments. If a bidder did not bid (or submitted a bid of zero) for the set of licenses that it is assigned, then no additional calculation is necessary, and the bidder will not have any additional assignment payment for that category in that assignment phase market. If, on the other hand, the bidder submitted a positive bid for the winning assignment, then the bidding system will calculate a type of 'second-price' assignment payment.[11]

The bidding system will apply bidder-optimal core prices and will use the "nearest Vickrey" approach to determine the assignment payments. In some cases, the second price (Vickrey price) may not be high enough to ensure that no bidder or group of bidders is willing to pay more for an alternative feasible assignment, and so an additional payment above Vickrey prices may be required. In the event that such a payment is required, the calculation of the additional payment to be paid by each bidder will be weighted based on the number of blocks won by the bidder for that category and assignment phase market.

The assignment payments will satisfy the following conditions:

First condition: Each assignment payment must be positive or zero and not more than the dollar amount of the winning assignment phase bid.

Second condition: The set of assignment payments must be sufficiently high that there is no bidder or group of bidders willing to pay more for an alternative feasible assignment. If there is only one set of assignment payments that satisfies the first two conditions, this determines the assignment payments.

Third condition: If there is more than one set of assignment payments that fulfill the first and second conditions, the set(s) of assignment payments minimizing the sum of assignment payments across all bidders is (are) selected. If there is only one set of assignment payments that satisfies these three conditions, this determines the assignment payments.

Fourth condition: If there is more than one set of assignment payments that satisfies the first three conditions, the set of assignment payments that minimizes the weighted sum of squares of differences between the assignment payments and the Vickrey prices will be selected. The weighting is relative to the number of blocks assigned to the bidder in the category and assignment phase market. This approach for selecting among sets of assignment payments that minimize the sum of assignment payments across bidders is referred to as the "nearest Vickrey" approach.

[11] In some cases, this may also be zero.

Section 6.1 describes how the Vickrey prices are calculated. Section 6.2 describes how payments are adjusted (if needed) to ensure that there is no bidder or group of bidders willing to pay more for an alternative feasible assignment. Section 6.3 provides an example.

6.1 Vickrey Price Calculation

For each bidder, the bidding system will determine the Vickrey price by re-solving the optimization problem of Section 5.1, but setting all bids of the bidder to zero while keeping the bids of every other bidder unchanged from the prior optimization, and calculating a hypothetical maximum sum of bid values from that optimization. The difference between the maximum sum of bid values associated with the actual optimization and the hypothetical maximum sum of bid values that would occur had that bidder provided all bids of zero is called the bidder's Vickrey discount. The Vickrey discount indicates the amount by which the bidder's winning bid amount exceeded the minimum amount it would have needed to bid to ensure the same winning assignment. The Vickrey price is calculated by subtracting the bidder's Vickrey discount from the bidder's actual bid amount.

Specifically, let $r(b)$ denote the maximum value attained by solving the optimization problem of Section 5.1 where the set of bid values is b. Let b_i^* be the bid amount of bidder i for the bidding option that it is assigned.

The Vickrey price of bidder i for a given category in a given assignment phase market is:

$$p_i^{Vickrey} = b_i^* - (\, r(b) - r(b_{i\to 0})\,)$$

where $b_{i\to 0}$ represents the set of bid values where the bid values of all bids of bidder i are set to zero (and the bid values of every other bidder are not changed).

6.2 Core Adjustments

An extra payment beyond the Vickrey prices is sometimes required in order to satisfy the second condition, which requires that the set of assignment payments is sufficiently high that there is no bidder or group of bidders prepared to pay more for an alternative feasible assignment. A bidder or group of bidders willing to pay more for an alternative feasible assignment is referred to as a blocking coalition of bidders. The group that is willing to pay the most forms the first blocking coalition. A blocking coalition is unblocked by increasing the assignment payments while ensuring that assignment payments are increased no more than necessary and that each bidder's assignment payment is less than or equal to the corresponding bid amount. After adjusting the assignment payments to unblock the first blocking coalition, additional blocking coalitions may exist that are each unblocked by again increasing the assignment payments until there is no bidder or group of bidders willing to pay more for an alternative feasible assignment. Each bidder's assignment payment is guaranteed to be at least its Vickrey price and no more than its bid amount for its assignment.

Assignment payments can be calculated iteratively via a core adjustment process. This process operates by starting with Vickrey prices and then by iteratively adjusting assignment payments until there is no bidder or group of bidders willing to pay more for an alternative feasible assignment. It does so by gathering pricing constraints from each blocking coalition and then satisfying the pricing constraints by selecting assignment payments that minimize the distance, weighted by the number of blocks being assigned to each bidder in the given category and assignment phase market, from Vickrey prices.

To formulate mathematically the core adjustment calculations, the following notation is used in addition to the notation of Section 5:

- b_i^* is the bid amount of bidder i for the bidding option that it is assigned (in the original assignment determination problem).

- p_i^n is the payment of bidder i for core adjustment in iteration n.
- C^n denotes the blocking coalition for iteration n.

In the first iteration, the payment of bidder i is set equal to the Vickrey price of bidder i, that is, $p_i^1 = p_i^{Vickrey}$.

Given a set of assignment payments for iteration n, calculate a reduced bid for each bidder and each of its bidding options by deducting the current surplus of the bidder ($b_i^* - p_i^n$) from the corresponding bid amount. Specifically, in iteration n, the reduced bid of bidder i for a bidding option $S \in F_i$ is:

$$b_i^n(S) = \max\{b_i(S) - (b_i^* - p_i^n), 0\}$$

Let C^n be the set of bidders with strictly positive bid amounts for the options they are assigned in the solution to the assignment determination problem for the set of reduced bids b^n in the optimization problem of Section 5.1. These bidders form the potential blocking coalition for iteration n. Among all potential blocking coalitions for iteration n, C^n is the one with the highest value, that is, $r(b^n)$.

If the value of the potential blocking coalition for iteration n does not exceed the sum of assignment payments for iteration n (that is, if $r(b^n) \leq \sum_{i \in N} p_i^n$), then there is no blocking coalition for iteration n, and p_i^n represents the assignment payment for bidder i. In this case, the assignment payment for bidder i is determined by rounding p_i^n up to the nearest integer, and no further calculations are required.

Otherwise, bidders in C^n do form a blocking coalition for iteration n, and the bidding system will calculate the updated set of assignment payments p_i^{n+1} as described below.

The bidding system will first calculate the minimum sum of assignment payments required to unblock all coalitions as of iteration n. This is done by solving the following optimization problem:

Minimize Sum of Assignment Payments (for third condition)

$$\mu^n = \min \sum_{i \in N} p_i$$

Subject to:

$\sum_{i \in N \setminus C^k} p_i \geq r(b^k) - \sum_{i \in C^k} p_i^k$	$\forall k \in \{1, \ldots, n\}$	(1)
$p_i \geq p_i^{Vickrey}$	$\forall i \in N$	(2)
$p_i \leq b_i^*$	$\forall i \in N$	(3)

Explanation of Constraints:

- Constraint (1) ensures that all coalitions are unblocked. In other words, for the blocking coalition of iteration k, the sum of assignment payments by bidders not in the coalition must be greater than or equal to the value of the coalition under the set of reduced bids for iteration k less the total iteration k payments of bidders in that coalition. There is one such constraint for each iteration.
- Constraint (2) requires that the price paid by each bidder be greater than or equal to the bidder's Vickrey price.
- Constraint (3) requires that the price paid by each bidder be less than or equal to the bidder's bid amount for its winning assignment.

The bidding system will then update the assignment payments of bidders such that they sum up to μ^n (the third condition).

If there is more than one set of assignment payments that sum up to μ^n, the set of assignment payments that minimizes the weighted sum of squares of differences between the assignment payments and the Vickrey prices will be selected. The weighting is relative to the number of blocks that the bidder won in the category and assignment phase market (the fourth condition).

The updated assignment payments p_i^{n+1} are calculated as the optimal solution to:

Minimize Distance Between Assignment Payments and Vickrey Prices (for fourth condition)

$$\min \sum_{i \in N} \frac{(p_i^{n+1} - p_i^{Vickrey})^2}{m_i}$$

Subject to:

$\sum_{i \in N \setminus C^k} p_i^{n+1} \geq r(b^k) - \sum_{i \in C^k} p_i^k$	$\forall k \in \{1, \ldots, n\}$	(1)
$p_i^{n+1} \geq p_i^{Vickrey}$	$\forall i \in N$	(2)
$p_i^{n+1} \leq b_i^*$	$\forall i \in N$	(3)
$\sum_{i \in N} p_i^{n+1} \leq \mu^n$		(4)

This quadratic problem minimizes the weighted sum of squares of differences between the updated assignment payments p_i^{n+1} and the Vickrey prices $p_i^{Vickrey}$, weighted by the number of blocks (m_i) won by each bidder.

Constraints (1) through (3) are the same as in the previous optimization.

Explanation of New Constraint:

Constraint (4) ensures that the sum of the updated payments is less than or equal to the minimum amount required to unblock all of the coalitions up to iteration n.

6.3 Assignment Payment Calculation Example

This section provides an example to illustrate how Vickrey prices and core adjustments are calculated in order to determine the assignment payments. In this example, in a PEA with ten Cat1 blocks, Bidder 1 won 2 blocks, and Bidders 2 and 3 won 4 blocks each.

The bids of each bidder are shown in Table 4.[12] The positive bid amounts are summarized below:

- Bidder 1 bids $1,000 on IJ.
- Bidder 2 bids $2,000 on CDEF.
- Bidder 3 bids $3,000 on GHIJ.

Assignment Determination: The sum of bid values is maximized when Bidder 1 is assigned AB, Bidder 2 is assigned CDEF, and Bidder 3 is assigned GHIJ. The value of this assignment is equal to $5,000.

Vickrey Prices: In this example, all Vickrey prices are $0 even though Bidder 3 is assigned two blocks for which Bidder 1 bid $1,000.

- The Vickrey price of Bidder 1 is equal to $0, because its bid value for its assignment is $0.
- To calculate the Vickrey price of Bidder 2, the bidding system solves the assignment determination optimization problem with all of the bids of Bidder 2 set to $0. The optimal value

[12] For simplicity, this example assumes that the bidders do not bid on all of their bid options.

of this optimization problem is $3,000, because Bidder 3 would be assigned GHIJ (with a value of $3,000) and the remaining blocks would be assigned to Bidders 1 and 2 (with a value of $0). Thus, the Vickrey discount for Bidder 2 is equal to $5,000-$3,000 = $2,000. The Vickrey price of Bidder 2 is then calculated as its bid amount for its assignment ($2,000) less its Vickrey discount ($2,000) and is therefore equal to $0.

- To calculate the Vickrey price of Bidder 3, the bidding system solves the assignment determination optimization problem with all of the bids of Bidder 3 set to $0. The optimal value of this optimization problem is $2,000, because Bidder 1 would be assigned AB (with a value of $0), Bidder 2 would be assigned CDEF (with a value of $2,000) and Bidder 3 would be assigned blocks GHIJ (with a value of $0). Note that it is not feasible to assign IJ to Bidder 1 and CDEF to Bidder 2, because then it would not be possible to assign contiguous spectrum to Bidder 3. Thus, the Vickrey discount for Bidder 3 is equal to $5,000-$2,000 = $3,000. The Vickrey price of Bidder 3 is then calculated as its bid amount for its assignment ($3,000) less its Vickrey discount ($3,000) and is therefore equal to $0.

<u>Iteration 1</u>. The next step is to determine whether there is a blocking coalition for iteration 1. To do this, the bidding system first calculates the surplus of each bidder for the first iteration as the bidder's bid amount for its assignment less its Vickrey price. The surplus is $0 for Bidder 1, $2,000 for Bidder 2, and $3,000 for Bidder 3. The bidding system then calculates a set of reduced bids for each bidder by subtracting the bidder's surplus from its actual bid amount for each bidding option. If the result for a bidding option is negative, the reduced bid amount is set to be equal to $0. The reduced bid amounts for the first iteration, b^1, are shown in Table 4. The bids of Bidder 1 are not reduced because Bidder 1 derives $0 surplus. The bids of Bidder 2 are reduced by up to $2,000 while ensuring that all bid values are non-negative. Similarly, the bids of Bidder 3 are reduced by up to $3,000 while ensuring that all bid values are non-negative.

The assignment determination problem is solved with the set of reduced bids for iteration 1. There is a blocking coalition (consisting of Bidder 1) with value $1,000. That is, Bidder 1 is willing to pay up to $1,000 more than the sum of Vickrey prices (which is $0) in order to be assigned IJ. The bidding system then calculates that the minimum sum of assignment payments to unblock this coalition is $\mu^1 = 1{,}000$. Because in this example Bidders 2 and 3 won the same number of blocks, the assignment payment of each of those bidders is incremented by the same amount, namely, by $500. Thus, the assignment payments for iteration 2 are: $p_1^2 = 0$ and $p_2^2 = p_3^2 = 500$.

<u>Iteration 2</u>. The bidding system checks whether there exists another blocking coalition by solving the assignment determination problem for the reduced set of bids b^2 shown in Table 4. The maximum sum of bids is equal to $1,000, which does not exceed the sum of assignment payments for this iteration. Thus, there does not exist a blocking coalition in iteration 2. This implies that the assignment payments are equal to p^2, that is, $0 for Bidder 1, $500 for Bidder 2, and $500 for Bidder 3.

Table 4: Assignment Payment Calculation Example

Bidders	*Bidder 1 (2 blocks)*	*Bidder 2 (4 blocks)*	*Bidder 3 (4 blocks)*
Bids	**AB: $0**	ABCD: $0	ABCD: $0
	CD: $0	**CDEF: $2,000**	CDEF: $0
	EF: $0	EFGH: $0	EFGH: $0
	GH: $0	GHIJ: $0	**GHIJ: $3,000**
	IJ: $1,000		
Vickrey prices (p^1)	$0	$0	$0
Bidder surplus at p^1	$0	$2,000	$3,000
Reduced bids Iteration 1 (b^1)	AB: $0	ABCD: $0	**ABCD: $0**
	CD: $0	CDEF: $0	CDEF: $0
	EF: $0	**EFGH: $0**	EFGH: $0
Coalition	GH: $0	GHIJ: $0	GHIJ: $0
$C^1 = \{1\}$	**IJ: $1,000**		
Adjusted payments p^2 (Assignment payments)	$0	$500	$500
Bidder surplus at p^2	$0	$1,500	$2,500
Reduced bids Iteration 2 (b^2)	**AB: $0**	ABCD: $0	ABCD: $0
	CD: $0	**CDEF: $500**	CDEF: $0
	EF: $0	EFGH: $0	EFGH: $0
No blocking	GH: $0	GHIJ: $0	**GHIJ: $500**
coalition	IJ: $1,000		

Note: This table does not list all the bidding options of a bidder. A bidding option is listed in this table only if it does not preclude other bidders from being assigned contiguous spectrum.

7 Information Policy

After the clock phase concludes but before bidding begins in the assignment phase, the bidding system will provide to each assignment phase bidder a menu of bidding options consisting of possible configurations of frequency-specific licenses on which it can bid. These bidding options will be consistent with the bidder's clock-phase winnings, but will not take into account the winnings of other bidders. The bidding system will also announce the order in which assignment rounds will take place and indicate which PEAs will be grouped together for bidding.

After each assignment round, the bidding system will inform each bidder of its own assignment and assignment payment for each assignment category for each assignment phase market assigned in the round. During the assignment phase, a bidder will not be provided any information about the assignments or the assignment payments of other bidders.

8 Final Auction Payment

This section describes how a bidder's final auction payment is calculated at the conclusion of the assignment phase.

This section uses the following notation:

- $d_{T,i,\{c,j\}}$ denotes the processed demand of bidder i for category c in PEA j after the final clock round. This is the number of blocks that the bidder won in category c in PEA j.
- $p_{T,\{c,j\}}$ denotes the posted price for category c in PEA j after the final clock round. This is the final clock phase price.
- R denotes the set of clock phase products, that is, all pairings of a PEA and a category.
- $AP_{i,k,c}$ denotes the assignment payment of bidder i for category c in assignment phase market k.
- AM denotes the set of assignment phase markets.
- SM denotes the set of assignment phase markets that consist of PEAs that are small markets.[13]
- $APM(k)$ denotes the set of PEAs in assignment phase market k.
- C_k denotes the categories in assignment phase market k. That is, $C_k = \{Cat1\}$ if assignment phase market k has a single category, and $C_k = \{Cat1, Cat2\}$ if assignment phase market k has two categories.
- BC_i denotes the bidding credit percentage of bidder i.
- FGP_i is the final gross payment of bidder i.
- FD_i is the final discount of bidder i.
- $GP_{i,k,c}$ denotes the gross payment of bidder i for assignment phase market k and category c. This is equal to the sum of the final clock phase prices for all licenses in the bidder's assignment *and* the bidder's assignment payment, for category c in assignment phase market k, that is,

$$GP_{i,k,c} = \sum_{j \in APM(k)} d_{T,i,\{c,j\}} \cdot p_{T,\{c,j\}} + AP_{i,k,c}$$

When all assignment rounds have completed, a bidder's final gross payment is determined by calculating the sum of the final clock phase prices across all licenses that it won and its assignment payments across all categories and assignment phase markets. Equivalently, the final gross payment of bidder i is equal to the sum of its gross payments across all assignment phase markets and categories:

$$FGP_i = \sum_{k \in AM} \sum_{c \in C_k} GP_{i,k,c}$$

For a bidder that does not qualify for a bidding credit, the *final auction payment* is equal to its final gross payment.

For a bidder that qualifies for a bidding credit, the *final auction payment* is equal to its final gross payment minus its final discount, that is, $FGP_i - FD_i$.

[13] Note that according to the rules for grouping PEAs into a single assignment phase market described in Section 2.1, *either* all PEAs in an assignment phase market are subject to the small market bidding credit cap *or* none of the PEAs in that assignment phase market are subject to the small market bidding credit cap.

If bidder i qualifies for the rural service provider bidding credit, then its final discount is:

$$FD_i = \min\{\$10 \text{ million}, BC_i \cdot FGP_i\}$$

That is, the bidder's final gross payment is multiplied by its bidding credit percentage and capped at $10 million.

If bidder i qualifies for the small business bidding credit, then its final discount is:

$$FD_i = \min\left\{\$25 \text{ million}, BC_i \cdot \sum_{k\in AM\setminus SM}\sum_{c\in C_k} GP_{i,k,c} + \min\left\{\$10 \text{ million}, BC_i \cdot \sum_{k\in SM}\sum_{c\in C_k} GP_{i,k,c}\right\}\right\}$$

This calculation first caps the discount from small markets at $10 million, then adds the discount from all other markets and caps the total at $25 million.

All the discount calculations described above will be rounded to the nearest dollar. Rounding will only be done at the very end of a given calculation, that is, after all summations, multiplications, and minimizations in a formula have been performed.

9 Calculations for Payment Information During Assignment Phase

While winning bidders will be expected to pay the final auction payment set forth above, the bidding system will show each bidder a running estimate of its auction payment obligations during the assignment phase. After each assignment round, each bidder will be shown its current total payment. A bidder that qualifies for a bidding credit will also be shown its current discount and total net payment.

This section uses the notation of Section 8.

9.1 Total Payment

The *total payment* of bidder i when A is the set of assignment phase markets for which an assignment has been processed is calculated as:

$$TP_i(A) = \sum_{r\in R} d_{T,i,r} \cdot p_{T,r} + \sum_{k\in A}\sum_{c\in C_k} AP_{i,k,c}$$

The first term, which represents the clock phase payment, is the product of the final clock phase price and the number of blocks won by the bidder, summed over all products. The second term is the sum of the bidder's assignment payments across all assignment phase markets that have been assigned so far. When an assignment has been processed for all assignment phase markets, the bidder's total payment is equal to its final gross payment.

9.2 Bidding Credit Discounts

All the discount calculations described in this section will be rounded to the nearest dollar. Rounding will only be done at the very end of a given calculation, that is, after all summations, multiplications, and minimizations in a formula have been performed.

9.2.1 Rural Service Provider Bidding Credit

Suppose that bidder i qualifies for the rural service provider bidding credit, and A is the set of assignment phase markets for which an assignment has been processed.

The current *uncapped discount* of bidder i is calculated as:

$$BC_i \cdot TP_i(A)$$

The current *discount* of bidder i is calculated as:

$$min\{\$10 \text{ million}, BC_i \cdot TP_i(A)\}$$

That is, the bidder's total payment is multiplied by its bidding credit percentage and capped at $10 million.

9.2.2 Small Business Bidding Credit

In this section, $TP_{i,SM}(A)$ is used to denote the part of the total payment of bidder i that relates to PEAs subject to the small market bidding credit cap, and $TP_{i,NSM}(A)$ is used to denote the part of the total payment that relates to PEAs not subject to the small market bidding credit cap.

Suppose that bidder i qualifies for the small business bidding credit, and A is the set of assignment phase markets for which an assignment has been processed.

The current *uncapped discount for small markets* only is:

$$BC_i \cdot TP_{i,SM}(A)$$

The current *uncapped discount* (across all markets) is:

$$BC_i \cdot TP_i(A)$$

The current *discount* (across all markets) is:

$$min\{\$25 \text{ million}, \ BC_i \cdot TP_{i,NSM}(A) + min\{\$10 \text{ million}, BC_i \cdot TP_{i,SM}(A)\}\}$$

This calculation first caps the discount from small markets at $10 million, then adds the discount from all other markets and caps the total at $25 million.

9.3 Total Net Payment

The *total net payment* is equal to the bidder's total payment minus its discount. Once all assignment rounds have been processed, a bidder's total net payment is equal to its final auction payment.

10 Per-License Price Calculations

While final auction payments for winning bidders will be calculated as in Section 8 above, with bidding credit caps and assignment payments applying on an aggregate basis rather than for individual licenses, the bidding system will also calculate a per-license price for each license. Such individual prices may be needed in the event that a licensee subsequently incurs license-specific obligations, such as unjust enrichment payments.

After the assignment phase, the bidding system will determine a net and gross price for each license that was won by a bidder by apportioning assignment payments and bidding credit discounts (only applicable for the net price) across all the licenses that the bidder won. To calculate the gross price for a given license, the bidding system will apportion the assignment payment for the corresponding category and assignment phase market to the licenses that the bidder is assigned in that category and market in proportion to the final clock phase prices of those licenses. To calculate the net price, the bidding system will first apportion any applicable bidding credit discounts to each category and assignment phase market in proportion to the gross payment for that category and market. Then, for each category and assignment phase market, the bidding system will apportion the assignment payment and the discount to licenses in proportion to the final clock phase prices of the licenses that the bidder is assigned in that category for that market.

This section uses the same notation as Section 8.

10.1 Apportioning Discounts to Each Category and Assignment Phase Market

This section describes how to apportion the bidder's final bidding credit discount across categories and assignment phase markets. Section 8 describes how the final discount of bidder i (FD_i) is calculated. Let $D_{i,k,c}$ denote the discount that is apportioned to category c and assignment phase market k.

If bidder i does not qualify for any bidding credit discount (and thus $FD_i = 0$), then $D_{i,k,c} = 0$ for all categories and assignment phase markets.

A bidder i that qualifies for the small business bidding credit is considered to have exceeded the small market bidding credit cap if $BC_i \cdot \sum_{k \in SM} \sum_{c \in C_k} GP_{i,k,c}$ rounded to the nearest integer is greater than $10 million.

If bidder i qualifies for the rural service provider bidding credit *or* if the bidder qualifies for the small business bidding credit and did not exceed the small markets cap, then:

$$D_{i,k,c} = \frac{GP_{i,k,c}}{FGP_i} \cdot FD_i$$

That is, the final discount is apportioned to categories in assignment phase markets proportionally to the bidder's gross payment in each category and assignment phase market. Recall that the final gross payment, FGP_i, is equal to the sum of gross payments across all categories and assignment phase markets (see Section 8).

For each category and assignment phase market, the calculation is rounded down to the nearest dollar. The slack due to rounding down is then distributed (one dollar at a time) to categories and assignment phase markets based on ascending order of gross payments. Ties are broken based on ascending lexicographic order of assignment phase market and category ID. The assignment phase market and category ID is defined as the PEA number of the lowest numbered PEA in the assignment phase market followed by the category (*e.g.*, PEA041-Cat1 or PEA125-Cat2).

If bidder i qualifies for the small business bidding credit and it exceeded the small market bidding credit cap, then:

- If $k \in SM$,

$$D_{i,k,c} = \frac{GP_{i,k,c}}{\sum_{k' \in SM} \sum_{c' \in C_{k'}} GP_{i,k',c'}} \cdot (\$10 \text{ million})$$

- If $k \notin SM$,

$$D_{i,k,c} = \frac{GP_{i,k,c}}{\sum_{k' \in AM \setminus SM} \sum_{c' \in C_{k'}} GP_{i,k',c'}} \cdot (FD_i - \$10 \text{ million})$$

That is, the $10 million discount that applies to small markets is apportioned to assignment phase markets that consist of PEAs subject to the small market bidding credit cap proportionally to the bidder's gross payments. The remaining discount (*i.e.*, $FD_i - \$10$ million) is apportioned among the assignment phase markets that consist of PEAs not subject to the small market bidding credit cap.

For each category and assignment phase market, the calculation is rounded down to the nearest dollar. The slack due to rounding down is then distributed (one dollar at a time) to categories and assignment phase markets based on ascending order of gross payments. Ties are broken based on ascending lexicographic order of assignment phase market ID.

In the case of a small business that exceeded the small market bidding credit cap, the apportioning of discounts and the distribution of any slack is done separately for the small markets and for the non-small markets.

10.2 Calculation of Gross Per-License Prices

Gross per-license prices are calculated by apportioning assignment phase payments to licenses in proportion to final clock phase prices.

Suppose that the bidder has been assigned a license in PEA j. Let k be the assignment phase market in which PEA j was assigned, that is, $j \in APM(k)$.

The gross price of a license in category c in PEA j is determined by the following formula:

$$p_{T,\{c,j\}} + \frac{p_{T,\{c,j\}}}{\sum_{j' \in APM(k)} d_{T,\{c,j'\}} \cdot p_{T,\{c,j'\}}} \cdot AP_{i,k,c}$$

That is, the assignment phase payment is apportioned to the licenses in the category and assignment phase market in proportion to the final clock phase price of each license. Note that if the bidder's assignment payment for the category and assignment phase market is zero, then the gross price of each license it is assigned in that market is simply the final clock phase price for that license.

Each license calculation is rounded down to the nearest dollar and then the slack due to rounding down is distributed to licenses (one dollar at a time) based on ascending order of final clock phase prices. Ties are broken based on ascending lexicographic order of license ID. License ID is defined as the PEA number followed by the letter representing the block (*e.g.*, PEA261-A or PEA001-C).

10.3 Calculation of Net Per-License Prices

Net per-license prices are calculated by apportioning assignment phase payments and discounts to licenses in proportion to final clock phase prices.

Suppose that the bidder has been assigned a license in PEA j. Let k be the assignment phase market in which PEA j was assigned, that is, $j \in APM(k)$.

The net price of a license in category c in PEA j is determined by the following formula:

$$p_{T,\{c,j\}} + \frac{p_{T,\{c,j\}}}{\sum_{j' \in APM(k)} d_{T,i,\{c,j'\}} \cdot p_{T,\{c,j'\}}} \cdot (AP_{i,k,c} - D_{i,k,c})$$

That is, for each category and assignment phase market, its assignment payment and its discount (see Section 10.1 for how the discount is determined) are apportioned to the licenses in that assignment phase market in proportion to the final clock phase price of each license. Note that if the bidder does not qualify for a bidding credit and its assignment payment in a category and assignment phase market is zero, then the net price of each license it is assigned in that category and market is simply the final clock phase price for that license.

Each license calculation is rounded down to the nearest dollar, and then the slack due to rounding down is distributed to licenses (one dollar at a time) based on ascending order of final clock phase prices. Ties are broken based on ascending lexicographic order of license ID.